The Incentive Theory of Matching: A Note

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1 Introduction

In a very nice paper, Brown, Merkl and Snower (2009)[1] describe their 'incentive theory' of matching as an alternative to (or micro-foundation for) most models in this literature, which assume an aggregate matching function as a reduced form way to describe search and matching frictions on the labor market. This is a very creative contribution. Moreover, the authors show that their model provides a better description of the data than the standard models along at least one crucial dimension: the volatility of job creation.

Apart from minor quibbles, I have two main problems with this paper. First, I think the focus on the Lucas critique is misplaced. As far as I am aware, noone would argue that the standard model with an aggregate matching function is *identical* to a model with heterogeneity. Rather, we think of the aggregate matching function as a reduced-form way to *approximately* capture the behavior of a model with heterogeneity. The exercise in the paper, what the authors call equivalence conditions, does not shed any light on this approximate similarity. The quantitative result of the paper, however, the finding that a calibrated version of the model with the incentive theory can replicate the observed volatility of unempoyment and job creation, is very interesting. Unfortunately, and this is my second criticism, the paper does not give any intuition for why this is the case.

In this note, I try to address both criticisms. First, and as a relatively minor point, I also argue that the paper focuses on the wrong simple case and show that steady state elasticities and perfectly flexible wages can be used to solve the model in closed form in order to get an intuition for the results. Then, using this approach, I show that the incentive theory generates amplification if and only if the distribution of idiosyncratic productivity shocks has sufficient mass around the hiring threshold. I also argue that the aggregate matching function is a good approximation of the incentive theory (and can also generate enough amplification) if we set the elasticity of the matching function with respect to unemployment sufficiently low.

[1] This note is based on the April 2009 version of the paper, titled "An Incentive Theory of Matching", which was distributed as IZA working paper No. 1512.

2 The standard search and matching model

In a standard model with matching function, the job finding rate μ_t is given by

$$\mu_t = \theta_t q\left(\theta_t\right) \tag{1}$$

where the function $q\left(\theta_t\right)$ is derived from the matching function and satisfies some regularity conditions . Labor market tightness θ_t is determined by the job creation equation,

$$\frac{\kappa}{q\left(\theta_t\right)} = \frac{1}{1+r} E_t J_{t+1} \tag{2}$$

where κ are vacancy posting costs, r is the discount rate and J_t is the value of having a filled job, which satisfies the following Bellman equation.

$$J_t = a_t - w_t + \frac{1-\lambda}{1+r} E_t J_{t+1} \tag{3}$$

This model can only be solved numerically. However, as argued among others by Mortensen and Nagypal (2007), the (deterministic) steady state provides a close approximation of the behavior of the model at business cycle frequencies, and can be solved in closed form.

$$\left(r+\lambda\right) J = \left(1+r\right)\left(a-w\right) \tag{4}$$

$$\frac{\kappa}{q\left(\theta\right)} = \frac{1}{1+r} J = \frac{a-w}{r+\lambda} \Leftrightarrow \theta = g\left(\frac{\left(r+\lambda\right)\kappa}{a-w}\right) \tag{5}$$

where $g\left(.\right)$ is the inverse of $q\left(.\right)$. Compared to equation (15) in the paper, which is derived under the much stronger assumption that the discount rate is infinite, this expression is equally simple, but provides a much better approximation of the full dynamics of the model.

We get the following (approximate) expression for the job finding rate,

$$\mu = \theta q\left(\theta\right) = \frac{\left(r+\lambda\right)\kappa}{a-w} g\left(\frac{\left(r+\lambda\right)\kappa}{a-w}\right) \tag{6}$$

which may be compared to the LHS of equation (17) in the paper.

3 Incentive theory

To be consistent with the steady state approximation of the standard model, I now work out the steady state approximation of the general form of the incentive theory in section 4, rather than the simple version in section 3, which assumes a zero discount factor. However, for now, I maintain the assumption that wages are exogenous as in the simple version.

From equation (21), with $\delta = 1 - r$, the expected net present value of profits of a match that accrue to the firm, after observing the idiosyncratic productivity shock ε in steady state are:

$$\pi = \frac{a - w - \varepsilon}{r} \tag{7}$$

The job offer rate is given by

$$\eta = P\left[\pi > h\right] = C_{\varepsilon}\left(a - w - rh\right) \tag{8}$$

and the firing rate is

$$\phi = P\left[\pi < -f\right] = 1 - C_{\varepsilon}\left(a - w + rf\right) \tag{9}$$

Notice that, although derived under a different simplifying assumption, these expressions are very similar to equations (3) and (5) for the simple model in the paper. Also notice that, as in the paper, if $h = f = 0$ then $\phi = 1 - \eta$.

From equations (28) and (30) we get the expected net present value of addition utility to the worker of being employed rather than unemployed, after observing the idiosyncratic disutility shock e in steady state.

$$V^N - V^U = \frac{w - b - e}{r\left(1 - \sigma - \mu\right) + \sigma + \mu} \simeq \frac{w - b - e}{r + \sigma + \mu} \tag{10}$$

Then, the job acceptance rate is

$$\alpha = P\left[V^N - V^U > 0\right] = C_e\left(w - b\right) \tag{11}$$

and the quit rate equals $\chi = 1 - \alpha$. These expressions are identical to (7) and (8) for the simple model in the paper.

The steady state job finding rate is given by $\mu = \eta\alpha$,

$$\mu = C_{\varepsilon}\left(a - w - rh\right) \cdot C_e\left(w - b\right) \tag{12}$$

and the separation rate equals $\sigma = \phi + \chi - \phi\chi$.

4 Simplifying the incentive theory

To make this model comparable with the standard model, in which separations are exogenous, I argue that we need to make separations exogenous. This can be done easily by shutting down the heterogeneity on the worker side, assuming firing costs are zero but that separations occur exogenously at rate λ. Effectively, this means that workers always accept any job offers they receive, so that all unemployment is involuntary. This assumption is not only realistic, but also coincides with what we assume in the standard model.

With exogenous separations and zero firing costs, the Bellman equation for profits (21) becomes

$$\pi_t = a_t - w_t - \varepsilon_t + \delta (1 - \lambda) E_t \pi_{t+1} \tag{13}$$

In steady state:

$$\pi = \frac{a - w - \varepsilon}{r + \lambda + r\lambda} \simeq \frac{a - w - \varepsilon}{r + \lambda} \tag{14}$$

The job finding rate equals the job offer rate and is given by

$$\mu = \eta = P [\pi > h] = C_\varepsilon (a - w - (r + \lambda) h) \tag{15}$$

We can obtain further simplification (and further comparability with the standard model) by realizing that, with exogenous separations, the hiring costs are no longer crucial. Assuming hiring costs are zero, we get

$$\mu = C_\varepsilon (a - w) \tag{16}$$

5 Wage determination

The paper treats wages as exogenous, at least in the simple version of the model that can be solved analytically. However, this assumption is not innocuous, because it means that the equivalence conditions have to hold for all w. I find it more insightful to instead assume a particular wage determination process. The wage determination process that makes the model easiest to solve in close form, and that can be used in both frameworks, is to assume that wages are proportional to productivity.

$$w = \beta a \tag{17}$$

This assumption is consistent with data on wages (Haefke, Sonntag and van Rens 2008), so that we can think of it as a good approximation of a calibrated version of a more general model, e.g. with Nash bargaining. It is also consistent with the calibration strategy in the current version of the paper, see p.19.

Under this assumption, the job finding rate under the standard model with a matching function is given by,

$$\mu = \frac{(r + \lambda)\kappa}{(1 - \beta) a} g \left(\frac{(r + \lambda)\kappa}{(1 - \beta) a} \right) \tag{18}$$

The job finding rate under the incentive theory is the following.

$$\mu = C_\varepsilon ((1 - \beta) a) \tag{19}$$

6 Job creation and the volatility puzzle

I now turn to the (approximate) predictions of both models for the volatility of job creation, by calculating the steady state elasticities of the job finding rate with respect to productivity, see e.g. Mortensen and Nagypal (2007). The cleanest way to compare the predictions of the two models is to compare the standard model with the simplified version of the incentive theory with exogenous separations and without hiring costs as derived above.

The standard model implies,

$$\frac{d\log\mu}{d\log a} = -1 - \frac{qg'(q)}{g(q)} \equiv \frac{1-\eta}{\eta} \tag{20}$$

where $\eta = -\frac{g(q)}{qg'(q)} = -\frac{\theta q'(\theta)}{q(\theta)}$, with $q = \frac{(r+\lambda)\kappa}{(1-\beta)a}$ and $\theta = g(q)$, is the elasticity of the matching function with respect to unemployment. Notice that this expression also follows from equation (9) in Haefke, Sonntag and van Rens (2008), which is derived in a slightly more general framework than here, if we assume that wages are proportional to productivity.

The incentive theory implies,

$$\frac{d\log\mu}{d\log a} = \frac{\nu^E C'_\varepsilon\left(\nu^E\right)}{C_\varepsilon\left(\nu^E\right)} \equiv \theta \tag{21}$$

where $\nu^E = (1-\beta)\,a$.

6.1 Equivalence with the matching function

As a first result, notice that the matching function provides a good description of the underlying heterogeneity in the incentive theory if $\eta = 1/(1+\theta)$ in steady state. There are three differences between this condition and the equivalence conditions in the paper.

1. I do not require the two models to be identically equal for all values of the state. It seems to me that the matching function is a good shortcut if the two models have *approximately* the same predictions.

2. I do not consider the derivative of the job finding rate with respect to the wage, but assume a (realistic) wage determination process and then consider the derivative with respect to productivity, taking into account the endogenous response of wages. I believe this results in a more realistic approximation of the models' predictions than the simple model in the paper.

3. Since in my simplified version of the incentive theory, the job finding rate no longer depends on the unemployment benefit, the derivative of the job finding rate with respect to that parameter is trivially the same (i.e. zero) in both models.

However, as I mentioned before, I do not see the 'Lucas critique' angle of this paper to be its main contribution. Instead, I now turn to exploring under which conditions the model can generate high volatility in job creation and thus address the unemployment volatility puzzle (Shimer 2005).

6.2 Amplification of productivity shocks

The standard model can generate an arbitrary amount of volatility if we set η low enough, as can be seen immediately from the steady state elasticities above. However, we do not consider this a realistic calibration given the estimates for η surveyed in Petrongolo and Pissarides (2001).

The incentive theory generates arbitrarily large volatility if θ is large enough. Given a value for $\nu^E = a - w = (1 - \beta)\, a$, which can easily be calibrated to average profits or average labor productivity and wages, this is a condition on the shape of the distribution of idiosyncratic productivity shocks. θ is infinity for a Dirac-δ distribution with all its mass at the threshold ν^E. On the other extreme, θ would be zero if the distribution of ε has zero mass at ν^E. Clearly, what matters is the mass of firms that are close to indifferent between hiring a worker or not. This is intuitive. Firms that are close to indifferent, would change their hiring decisions depending on small variations in the level of productivity.

In my opinion, a careful calibration of θ, is the most important thing that needs to be added to this paper. Currently, the distribution of ε, and therefore θ, is calibrated in a way that does not do justice to its importance for the volatility results. Idiosyncratic productivity shocks are assumed to be logistically distributed, which limits the number of paramters to be calibrated to two. Then, the two parameters of this distribution are calibrated to the elasticity of wages with respect to productivity and the steady state of worker flows. This calibration strategy relies heavily on the arbitrary assumption on the functional form of the distribution, which imposes strong restrictions on the predictions of the model for steady states and volatility. I think the calibration target should be informative about the shape of the distribution in the region that matters for the main result, i.e. on the mass of 'marginal' firms.

7 Extensions

7.1 Hiring costs

In the full incentive theory, there are costs that firms need to pay to hire a worker. These hiring costs provide an additional source of amplification. To see this, consider the expression for the job finding rate with non-zero hiring costs h (but still with exogenous separations) above and substitute the process for wage determination.

$$\mu = C_{\varepsilon}\left(\left(1-\beta\right)a - \left(r+\lambda\right)h\right) \tag{22}$$

This expression implies the following steady state elasticity with respect to productivity.

$$\frac{d\log\mu}{d\log a} = \theta\frac{\left(1-\beta\right)a}{\left(1-\beta\right)a - \left(r+\lambda\right)h} \tag{23}$$

Now, by setting h high enough, we can get arbitrarily much amplification out of this model even if θ is low. The intuition for this result is that hiring costs reduce the surplus of the match and therefore profits accruing to firms, thus amplifying the proportional effect of changes in productivity on profits.

However, exactly the same mechanism operates in the standard model with a matching function as well, see e.g. Pissarides (2009), section 5, or Mortensen and Nagypal (2007), section 3.3. In this model, in the presence of hiring costs the job creation equation becomes,

$$\frac{\kappa}{q\left(\theta\right)} = \frac{1}{1+r}J - h = \frac{a-w}{r+\lambda} - h \Leftrightarrow \theta = g\left(\frac{\left(r+\lambda\right)\kappa}{a-w-\left(r+\lambda\right)h}\right) \tag{24}$$

so that the job finding rate is given by

$$\begin{aligned}
\mu &= \left(\frac{\left(r+\lambda\right)\kappa}{a-w-\left(r+\lambda\right)h}\right) g\left(\frac{\left(r+\lambda\right)\kappa}{a-w-\left(r+\lambda\right)h}\right) && (25)\\
&= \left(\frac{\left(r+\lambda\right)\kappa}{\left(1-\beta\right)a-\left(r+\lambda\right)h}\right) g\left(\frac{\left(r+\lambda\right)\kappa}{\left(1-\beta\right)a-\left(r+\lambda\right)h}\right) && (26)
\end{aligned}$$

Then,

$$\frac{d\log\mu}{d\log a} = \frac{1-\eta}{\eta}\frac{q}{q-h} = \frac{1-\eta}{\eta}\frac{\left(1-\beta\right)a}{\left(1-\beta\right)a-\left(r+\lambda\right)h} \tag{27}$$

so that the amplification coming from hiring costs is the same in both models.

7.2 Endogenous separations

In the incentive theory model in the paper, we get even more amplification from the endogenous separations. In the full model, using the expression for the steady state job finding rate and the wage determination mechanism from above,

$$\mu = C_{\varepsilon}\left(\left(1-\beta\right)a - rh\right)\cdot C_{e}\left(\beta a - b\right) \tag{28}$$

so that the steady state elasticity is given by,

$$\frac{d\log\mu}{d\log a} = \theta\frac{\left(1-\beta\right)a}{\left(1-\beta\right)a-\left(r+\lambda\right)h} + \frac{\nu^{I}C_{e}\left(\nu^{I}\right)}{C_{e}\left(\nu^{I}\right)}\frac{\beta a}{\beta a-b} \tag{29}$$

where $\nu^I = \beta a - b$. I only want to note that we should compare apples to apples, not oranges, so that the standard model (with exogenous separations) should be compared to the incentive theory with exogenous separations. Without going through the derivations, my guess is that if we would extend the standard model with endogenous separations, e.g. as in Mortensen-Pissarides (1994), then we could show a similar type of equivalence as for job creation.

8 Conclusion

Concluding, I find this a very thought-provoking paper. If nothing else, it provides us with a much better idea about how to calibrate the aggregate matching function, akin to how Rogerson and Wallenius (2007) or Gourio and Noual (2007) calibrate the aggregate (Frisch) elasticity of labor supply. But it seems to me that the current version of the paper focuses on the wrong implications of the theory, and that a thoughtful evaluation of the main contribution is missing. Therefore, I recommend the authors to:

1. Make the incentive theory better comparable to the standard model with an aggregate matching function by making separations exogenous and setting hiring costs to zero.

2. Drop the focus on the Lucas critique of the aggregate matching function.

3. Add a careful discussion about how to calibrate the shape of the distribution and some evidence that the actual shape generates enough volatility (or not).

4. Think about how to reconcile the predictions of the incentive theory with the estimates of the matching function. If the world is truly described by the incentive theory, then how do we explain the estimates in Petrongolo and Pissarides?