1 Preface

This note is *not* a criticism of the main methodological contribution of the paper by Gertler and Trigari, which is to introduce wage rigidity into a DSGE model with frictional labor markets in a tractable manner. Rather, I argue that their model cannot be used to make statements about the performance of search and matching models to describe (un)employment fluctuations, because their model is not a search model. Therefore, it is a critique of their substantive contribution.

The idea is not mine, but was brought up by Larry Christiano in a conversation at Northwestern. I wrote the note just to check if his intuition is right (it is!) and to be able to explain it better to others.

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2 Why Gertler-Trigari is not a search model

The suggestion in the paper is that the Gertler-Trigari (GT) model is a 'modified' search and matching model, that uses quadratic adjustment costs, rather than the more conventional linear vacancy posting costs. This suggestion is misleading.

The core of a search model is a 'congestion externality': a firm posting a vacancy does not take into account that this additional vacancy increases aggregate labor market tightness and thus makes it harder for other firms to hire workers for their vacancies. This congestion externality is described by a matching function, which gives rise to worker a finding probability for firms (and a job finding probability for workers) that depends on aggregate labor market tightness. The congestion externality is important for (un)employment fluctuations because the job finding probability enters the law of motion for unemployment:

$$u_{t+1} = u_t + \delta\left(1 - u_t\right) - p\left(\theta_t\right) u_t$$

where δ is the separation rate ($1 - \rho$ in GT) and $p\left(\theta_t\right)$ is the job finding probability (s_t in GT).

What I argue here, is that in the GT model, the matching function does not affect (un)employment fluctuations and thus that that there is no congestion externality. In that sense. the model is not a search and matching model, but a model with convex adjustment costs in employment. We can think of the model as a partial equilibrium version of a search model, in which there is no general equilibrium feedback effect of vacancy posting on the profitability of a vacancy.

Given that the congestion externality dampens the response of aggregate vacancy creation to changes in productivity shocks, it is likely that a model with convex adjustment costs displays less of an unemployment volatility puzzle than a model with search frictions. Thus, it must be that the GT model, even without wage rigidity exhibits more unemployment volatility than a search model.

2.1 Argument 1 (firms' problem)

Adjustment costs are specified over the hiring rate $x_t(i)$, i.e. a firm's flow profits are given by (see equation 8 in the paper).[1]

$$y_t(i) - w_t(i) n_t(i) - z_t k_t(i) - \frac{\kappa}{2} x_t(i)^2 n_t(i)$$

Profits depend on labor market flows, because the law of motion for employment depends on the aggregate worker finding rate q_t, see equation 7).

$$n_{t+1}(i) = \rho n_t(i) + q_t v_t(i)$$

where we can think of the firm choosing vacancies $v_t(i)$, taking the aggregate worker finding probability q_t as given. However, using the definition of the hiring rate, equation 6,

$$x_t(i) = \frac{q_t v_t(i)}{n_t(i)}$$

we can rewrite the law of motion for employment as follows,

$$n_{t+1}(i) = (\rho + x_t(i)) n_t(i) \tag{7'}$$

and think of the firm as directly choosing $x_t(i)$.

This explains why in the Euler equation for the hiring rate, equation 12, no aggregate labor market variables appear anymore. Euler equation (12), combined with the law of motion for employment (7') above, plus a transversality condition, completely determine employment $n_t(i)$ and the hiring rate $x_t(i)$. And since neither equation depends on aggregate labor market variables, it must be that employment fluctuations of firm i are independent of aggregate labor market conditions. Thus there is no congestion externality.

2.2 Argument 2 (complete log-linearized model)

An alternative way to see the same thing, is to start from the full log-linearized model in appendix C. The goal here is to show that employment $\hat{n}_t$ is not affected by the specification of the matching function E3, and in particular does not depend on its elasticity σ. To show this, I rewrite equations E4 (the law of motion for employment) and E18, without using the matching function E3.

The law of motion for employment (E4), can be rewritten in terms of the hiring rate as follows.

$$\begin{aligned}
\hat{n}_{t+1} &= \rho \hat{n}_t + (1-\rho)\hat{m}_t \\
&= \rho \hat{n}_t + (1-\rho)(\hat{q}_t + \hat{v}_t) \quad \text{using E5} \\
&= \rho \hat{n}_t + (1-\rho)(\hat{x}_t + \hat{n}_t) \quad \text{using E9} \\
&= \hat{n}_t + (1-\rho)\hat{x}_t \qquad \text{(E4')}
\end{aligned}$$

[1] Equation numbers refer to the August 2006 NBER working paper version of the paper.

This equation is of course just the log-linearization of the aggregate version of the law of motion for an individual firm (7') derived above.

Equation E18 features the aggregate job finding probability $\hat{s}_t$. Again using only definitional equations, we get

$$\begin{aligned}
\hat{s}_t &= \hat{m}_t - \hat{u}_t \quad \text{by E6} \\
&= \hat{q}_t + \hat{v}_t - \hat{u}_t \quad \text{using E5} \\
&= \hat{x}_t + \hat{n}_t - \hat{u}_t \quad \text{using E9} \\
&= \hat{x}_t + \hat{n}_t + \frac{n}{u}\hat{n}_t \quad \text{using E7} \\
&= \hat{x}_t + \frac{1}{1-n}\hat{n}_t \qquad \text{(E18a)}
\end{aligned}$$

Substituting (E18a) into (E18) we get equation E18', in which no aggregate labor market variables other than $\hat{x}_t$ and $\hat{n}_t$ appear.

Equations E1, E2, E4', E8, E10-E17, E18', E19 and E20 fully describe the equilibrium of the log-linearized model. No labor market variables other than $\hat{x}_t$ and $\hat{n}_t$ appear in these equations, nor does the parameter σ of the matching function. Thus, also in the full model, employment fluctuations (as well as unemployment fluctuations by equation E7) do not depend on the matching function and thus there is not congestion externality.

Of course the model does have predictions for vacancies and worker flows. Equations E3, E5, E6 and E9 are four (static) equations that determine $\hat{m}_t$, $\hat{v}_t$, $\hat{q}_t$ and $\hat{s}_t$ given $\hat{n}_t$, $\hat{u}_t$ and $\hat{x}_t$. For these variables, the matching function matters. It is irrelevant however, for $\hat{n}_t$ and $\hat{u}_t$, which are arguably the variables of most interest for the unemployment volatility puzzle.