Labor Markets and Business Cycles

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Overview of Lectures

1. The Labor Wedge
2. Benchmark Search Model
3. Capital
4. Rigid Wages

The Labor Wedge

Representative Agent Model

- representative household
  - consumes
  - supplies labor
  - owns firms and government bonds
- representative firm
  - produces output (single consumption and capital good)
  - demands labor
  - buys and sells capital
- government: sets taxes, transfers, and spending, and issues bonds
- perfect competition

States and History

- time is $t = 0, 1, 2, \ldots$
- state of the economy at t is s_t
- history of the economy at t is $s^t \equiv \{s_0, s_1, \ldots, s_t\}$
    - productivity
    - government spending
    - distortionary tax rates

      $\vdots$
- $\Pi(s^t)$ is time-0 probability of history s^t

Household Problem

- household chooses $\{c(s^t), h(s^t)\}$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left(\log c(s^t) - \frac{\gamma \varepsilon}{1+\varepsilon} h(s^t)^{\frac{1+\varepsilon}{\varepsilon}} \right),$$

subject to

$$a_0 = \sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t) \big(c(s^t) - (1 - \tau(s^t)) w(s^t) h(s^t) - T(s^t) \big)$$

taking a_0 and $\{q_0(s^t), w(s^t), \tau(s^t), T(s^t)\}$ as given

Firm Problem

- firm chooses $\{h^d(s^t), k(s^{t+1})\}$ to maximize

$$\sum_{t=0}^{\infty}\sum_{s^t} q_0(s^t)\left(z(s^t)k(s^t)^\alpha h^d(s^t)^{1-\alpha} + (1-\delta)k(s^t) - k(s^{t+1}) - w(s^t)h^d(s^t)\right)$$

  taking $k_0 = k(s^0)$ and $\{q_0(s^t), w(s^t)\}$ as given

- call the value of the firm $J(s^0)$

Government

- government faces a budget constraint

$$b_0 = \sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t) \big(\tau(s^t) w(s^t) h(s^t) - g(s^t) - T(s^t) \big)$$

Market Clearing

- labor market clearing: $h(s^t) = h^d(s^t)$ for all t
- goods market clearing:

$$k(s^{t+1}) = z(s^t)k(s^t)^{\alpha}h^d(s^t)^{1-\alpha} + (1-\delta)k(s^t) - c(s^t) - g(s^t)$$

  for all t
- capital market clearing: $a(s^t) = J(s^t) + b(s^t)$ for all t
  - $a(s^t)$: household assets in history s^t
  - $J(s^t)$: value of firm in history s^t
  - $b(s^t)$: value of government debt in history s^t
  - this is implied by the other equations

Equilibrium

- a_0, b_0, k_0 and $\{c(s^t), h(s^t), h^d(s^t), k(s^t), q_0(s^t), w(s^t), \tau(s^t), T(s^t), g(s^t)\}$ s.t.:
  - household problem is solved
  - firm problem is solved
  - government budget constraint is satisfied
  - labor and goods markets clear

Marginal Rate of Substitution Equals Wage

- household solves

$$\max_{\{c(s^t),h(s^t)\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left(\log c(s^t) - \frac{\gamma\varepsilon}{1+\varepsilon} h(s^t)^{\frac{1+\varepsilon}{\varepsilon}} \right)$$

$$\text{s.t. } a_0 = \sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t) \big(c(s^t) - (1-\tau(s^t)) w(s^t) h(s^t) - T(s^t) \big)$$

- first order conditions:
  - $c(s^t)$: $\dfrac{\beta^t \Pi(s^t)}{c(s^t)} = \lambda q_0(s^t)$
  - $h(s^t)$: $\beta^t \Pi(s^t) \gamma h(s^t)^{\frac{1}{\varepsilon}} = \lambda q_0(s^t)(1-\tau(s^t)) w(s^t)$

Marginal Rate of Substitution Equals Wage

- household solves

$$\max_{\{c(s^t),h(s^t)\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left(\log c(s^t) - \frac{\gamma\varepsilon}{1+\varepsilon} h(s^t)^{\frac{1+\varepsilon}{\varepsilon}} \right)$$

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  - $h(s^t)$: $\beta^t \Pi(s^t) \gamma h(s^t)^{\frac{1}{\varepsilon}} = \lambda q_0(s^t)(1-\tau(s^t)) w(s^t)$

$$w(s^t) = \frac{\gamma c(s^t) h(s^t)^{\frac{1}{\varepsilon}}}{1-\tau(s^t)}$$

Marginal Product of Labor Equals Wage

- firm solves

$$\sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t)\big(y(s^t) + (1-\delta)k(s^t) - k(s^{t+1}) - w(s^t)h^d(s^t)\big)$$

where $y(s^t) = z(s^t)k(s^t)^\alpha h^d(s^t)^{1-\alpha}$

- first order conditions:
  - $h^d(s^t)$: $q_0(s^t)\big((1-\alpha)z(s^t)k(s^t)^\alpha h^d(s^t)^{-\alpha} - w(s^t)\big) = 0$

Marginal Product of Labor Equals Wage

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$$w(s^t) = \frac{(1-\alpha)y(s^t)}{h^d(s^t)}$$

Labor Market Clearing

$$h(s^t) = h^d(s^t)$$

The Labor Wedge

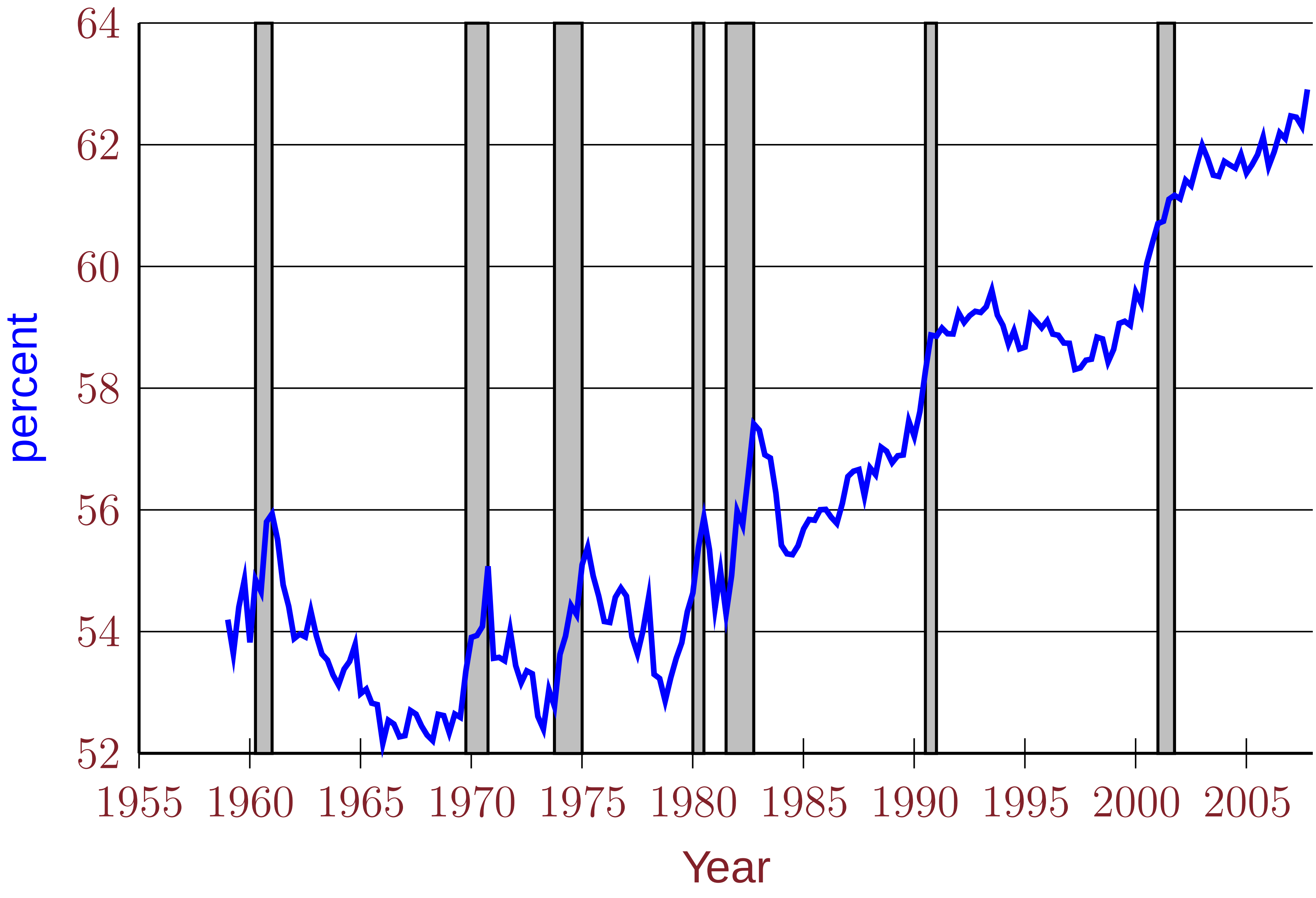
$$\tau(s^t) = 1 - \frac{\gamma}{1-\alpha}\left(\frac{c(s^t)}{y(s^t)}\right) h(s^t)^{\frac{1+\varepsilon}{\varepsilon}}$$

Measurement

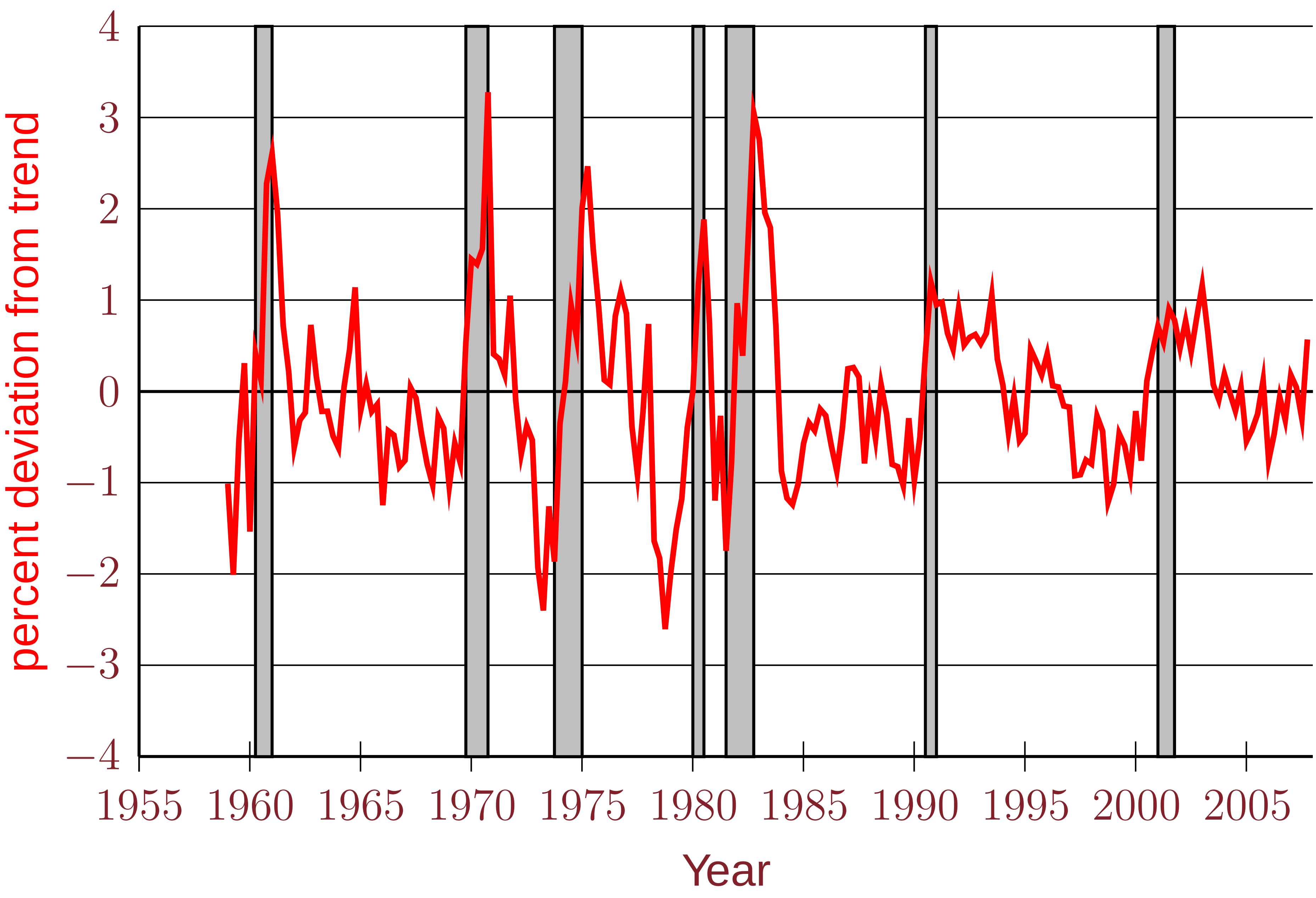
Measurement

- consumption-output ratio
  - consumption: nominal PCE including nondurables and services
  - output: nominal GDP
- hours:
  - Prescott, Ueberfeldt, and Cociuba (2008)
  - total hours worked from CPS, plus estimate of military
    - number of civilians at work times average hours worked
    - estimate 40 hours per week for the military
  - deflate by noninstitutional population aged 16 to 64

Consumption-Output Ratio



Consumption-Output Ratio



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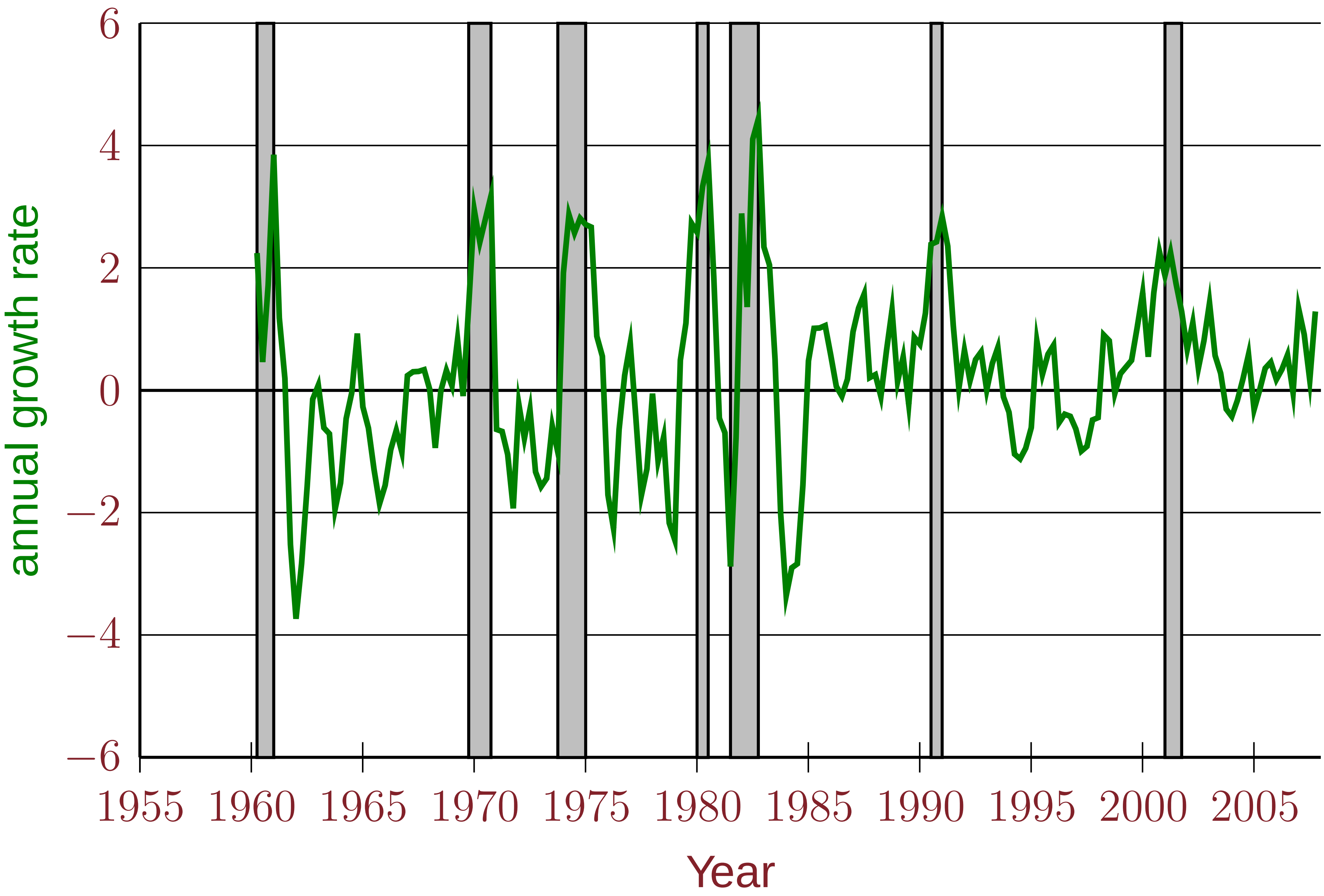

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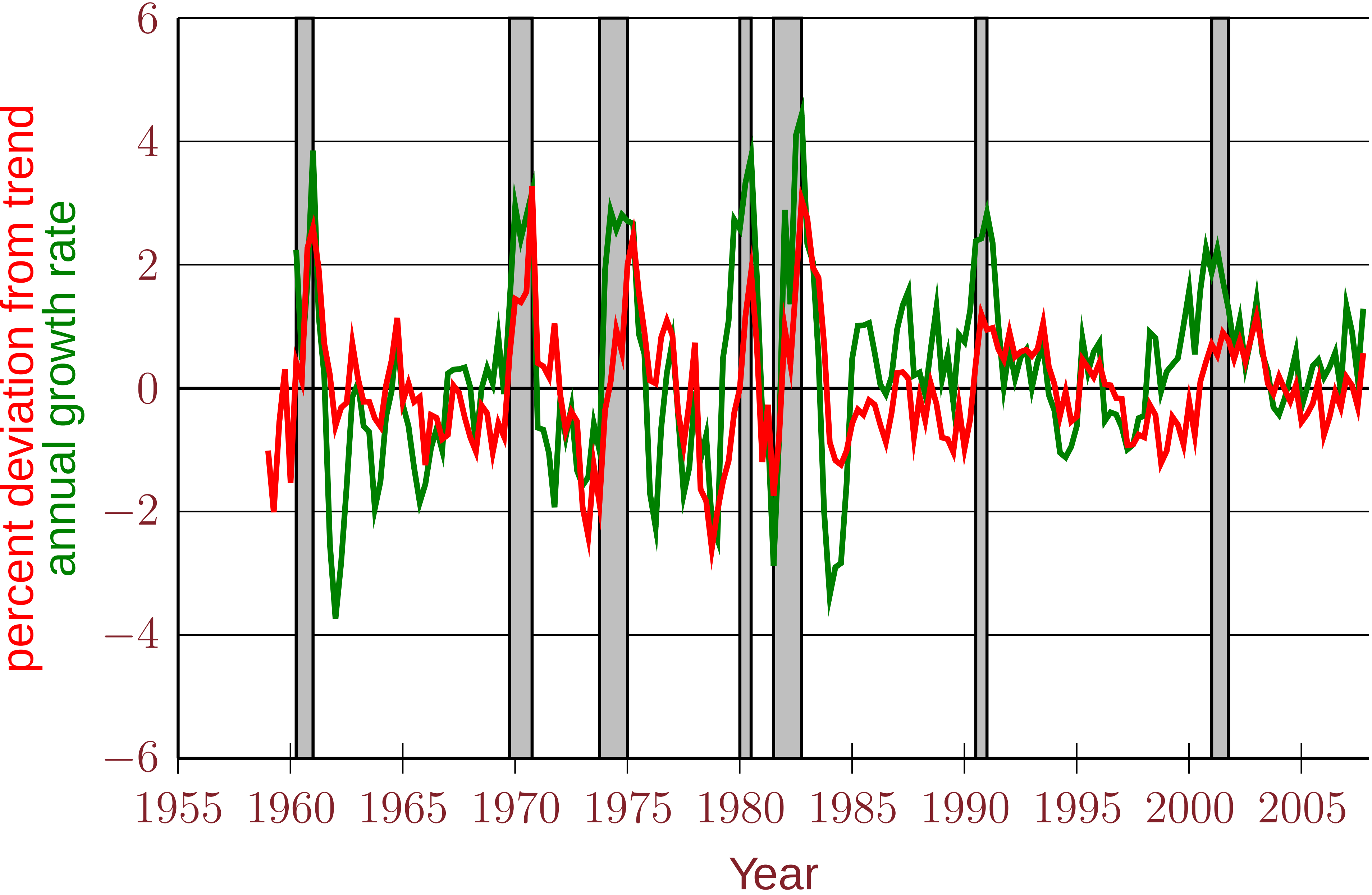
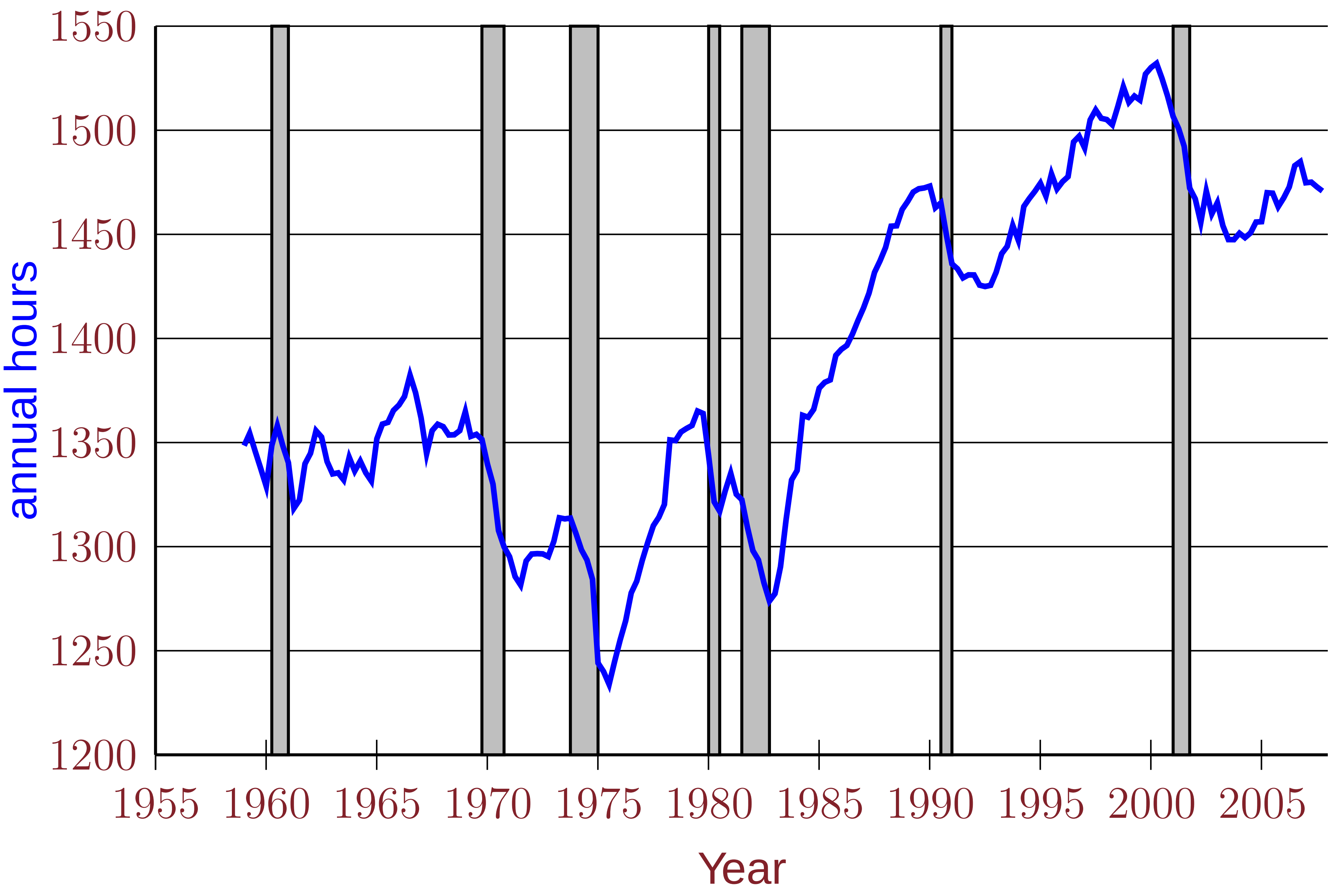


Hours Worked



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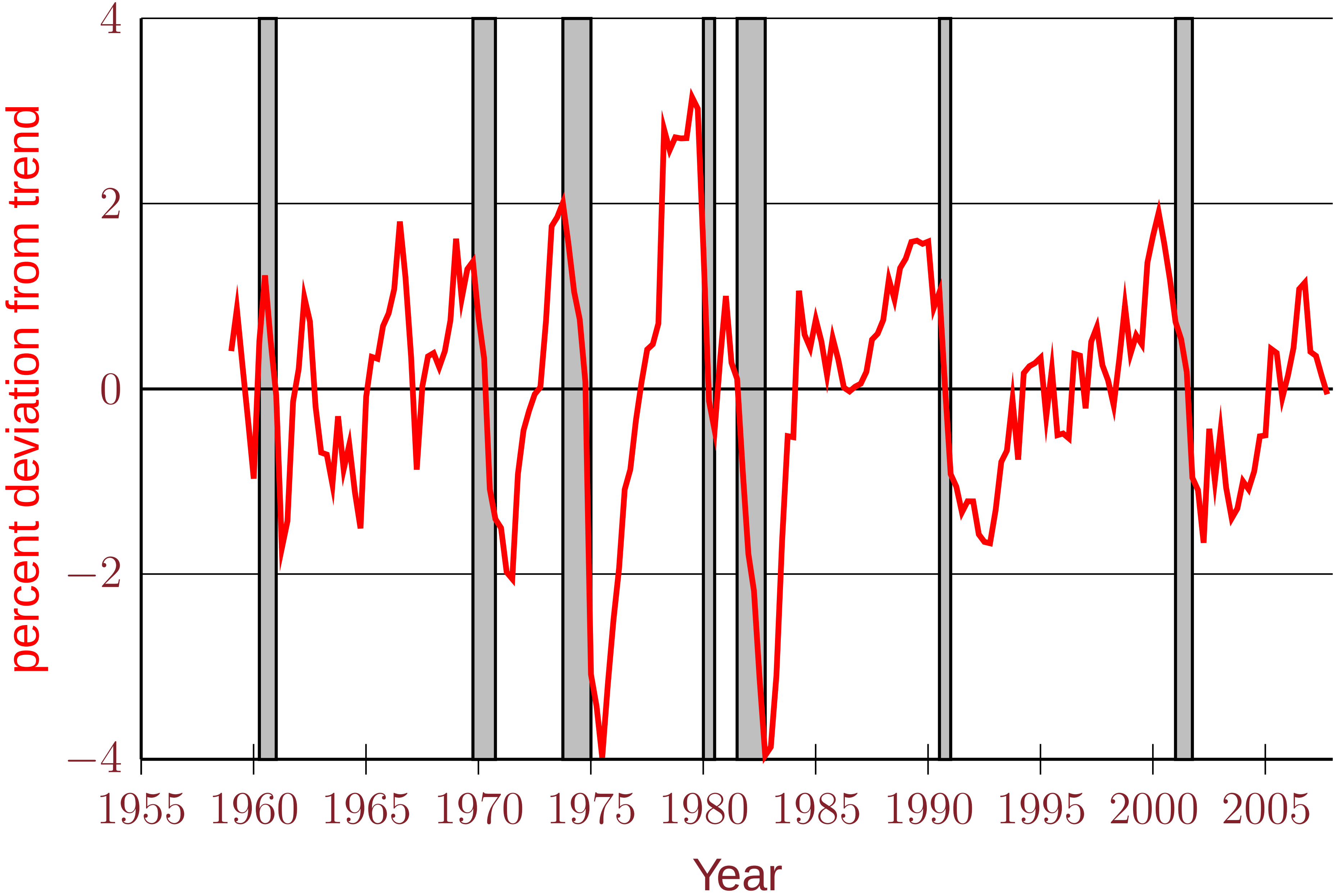

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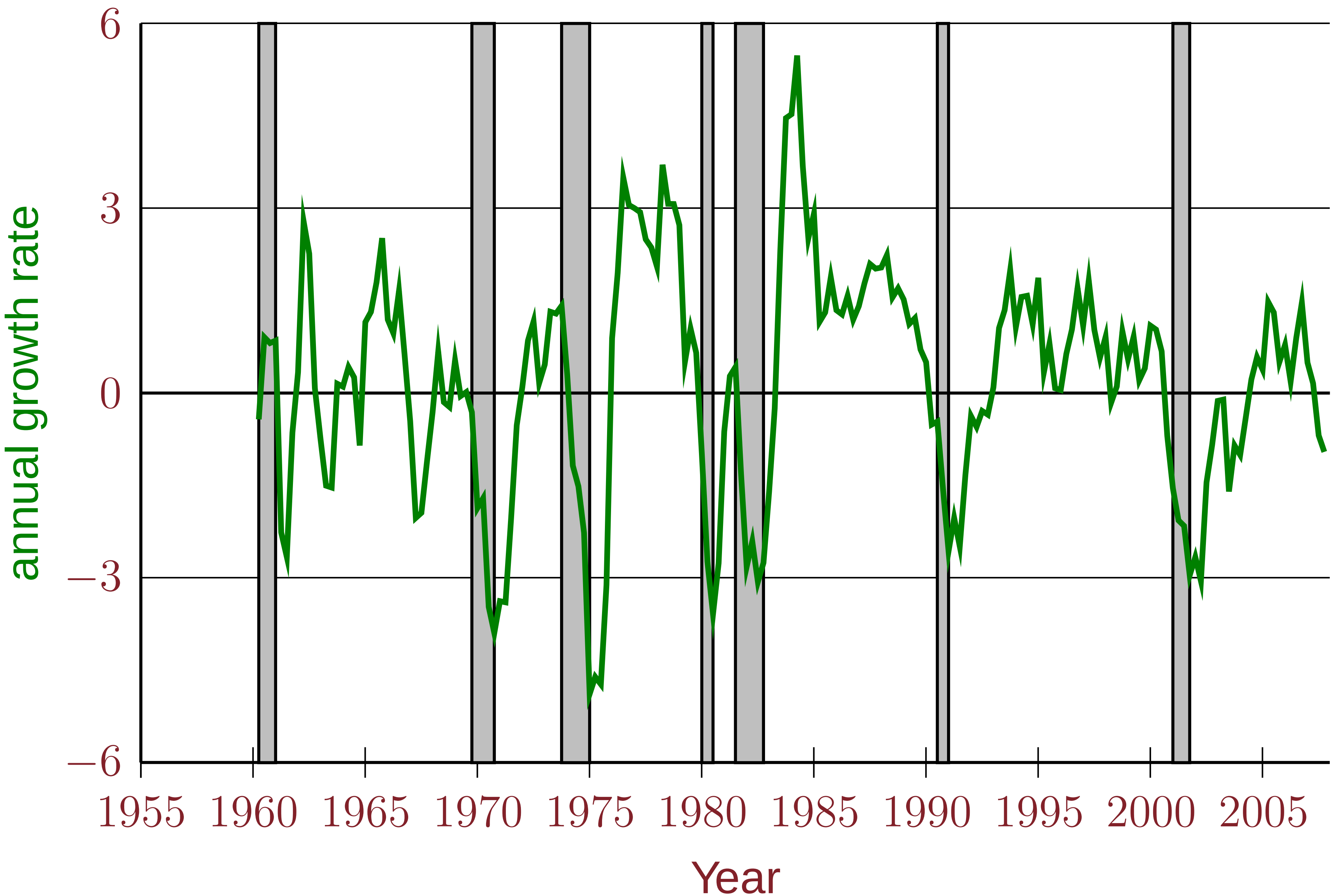

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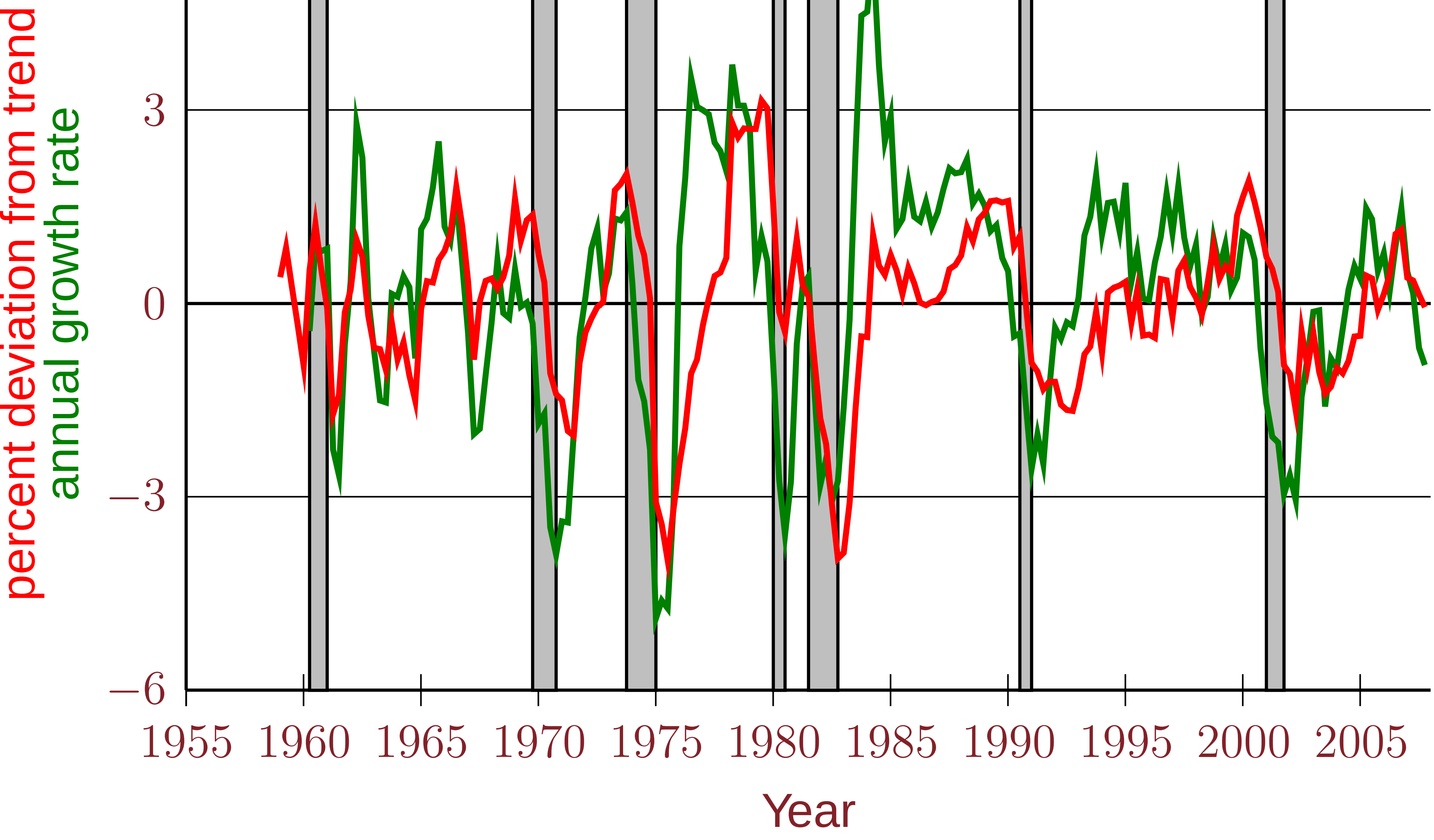


Comovement of c/y and h

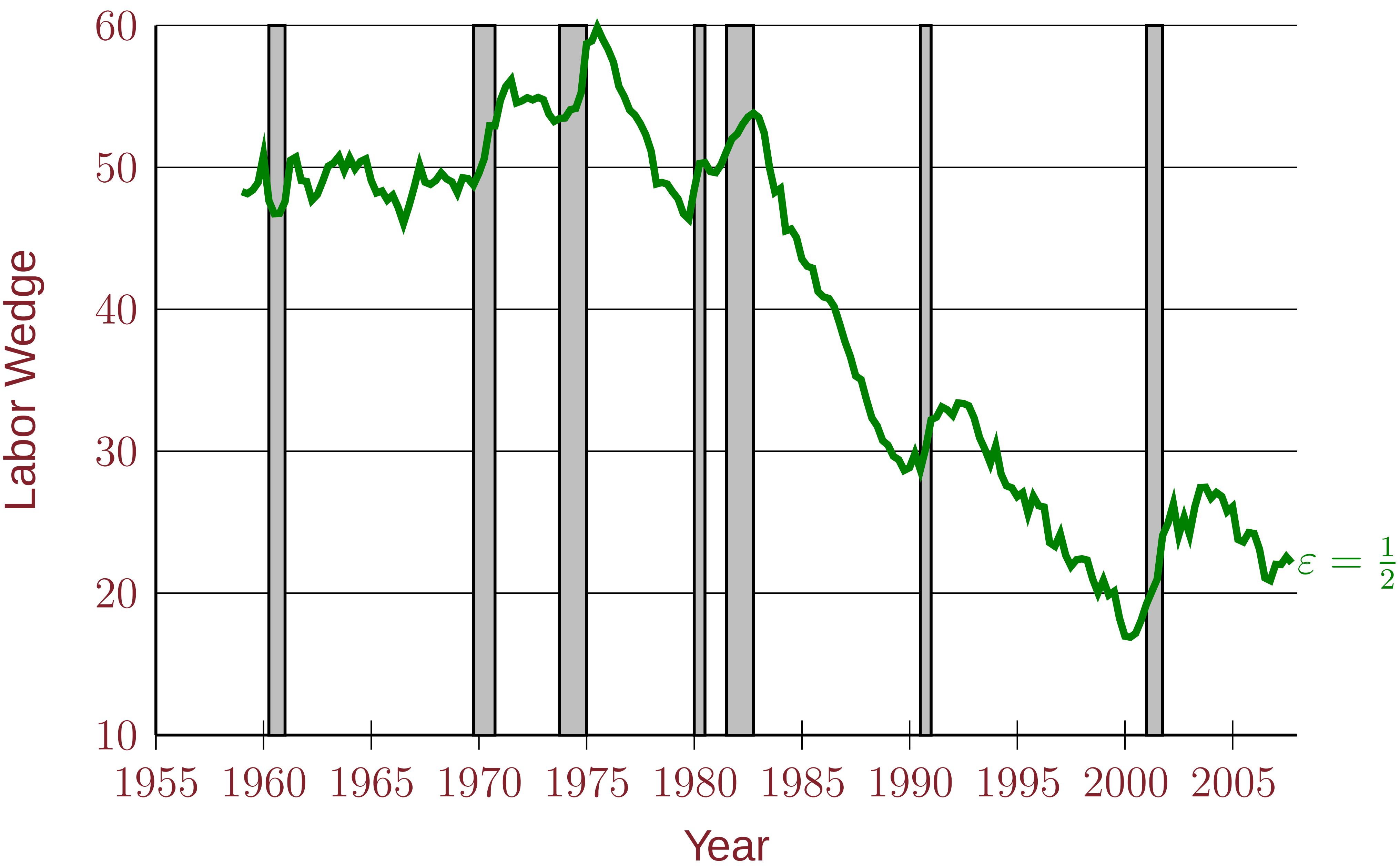
	s.d. c/y	s.d. h	relative s.d.	correlation
detrended	0.010	0.013	1.32	-0.663
annual growth	0.015	0.018	1.25	-0.571

Measuring the Labor Wedge

$$\tau(s^t) = 1 - \frac{\gamma}{1-\alpha}\left(\frac{c(s^t)}{y(s^t)}\right) h(s^t)^{\frac{1+\varepsilon}{\varepsilon}}$$

- use data for c/y and h
- use different values of ε
- set $\gamma/(1-\alpha)$ so $\tau = 0.4$ on average

Labor Wedge



Labor Wedge
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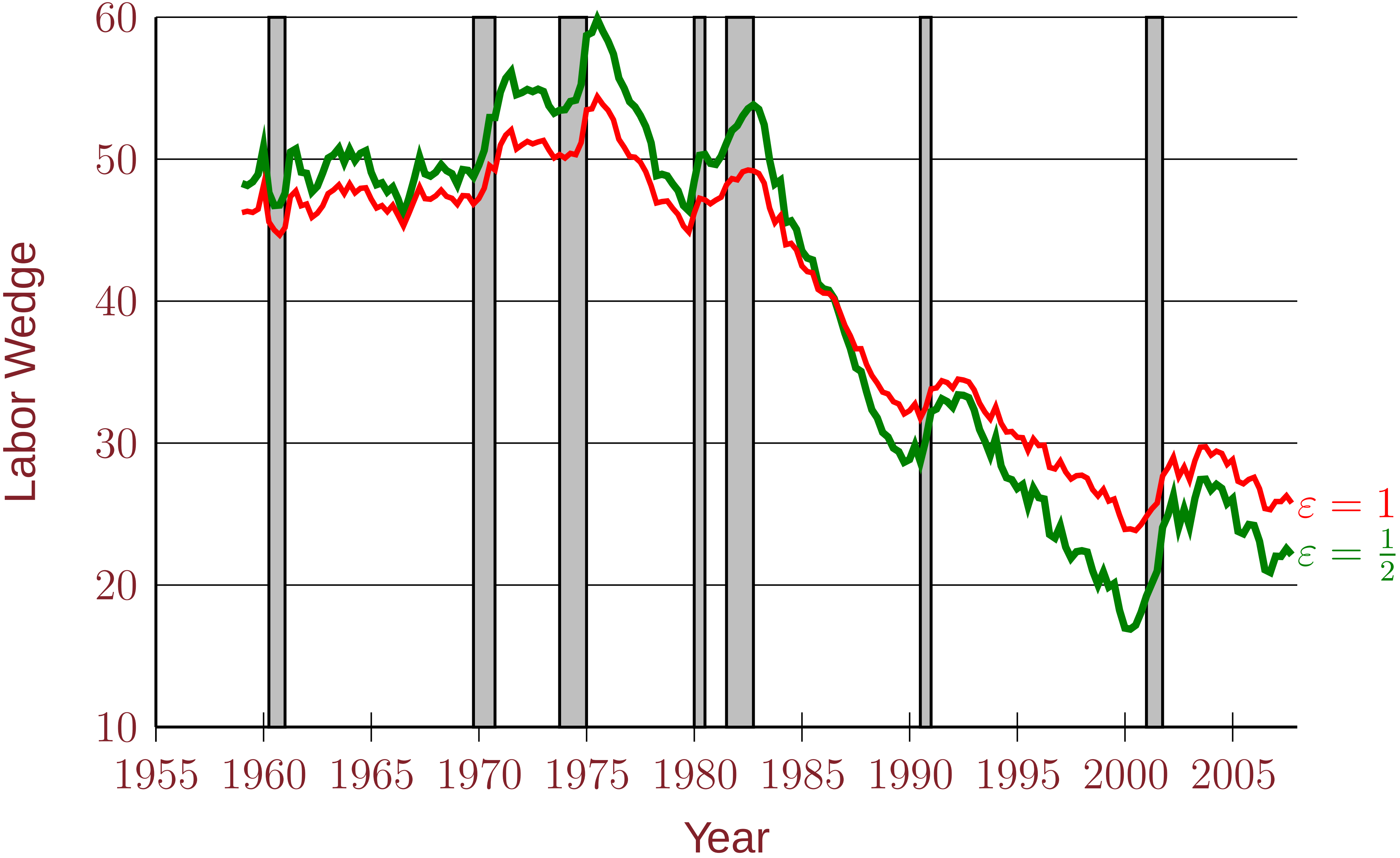
Year
$\varepsilon = \frac{1}{2}$

Labor Wedge



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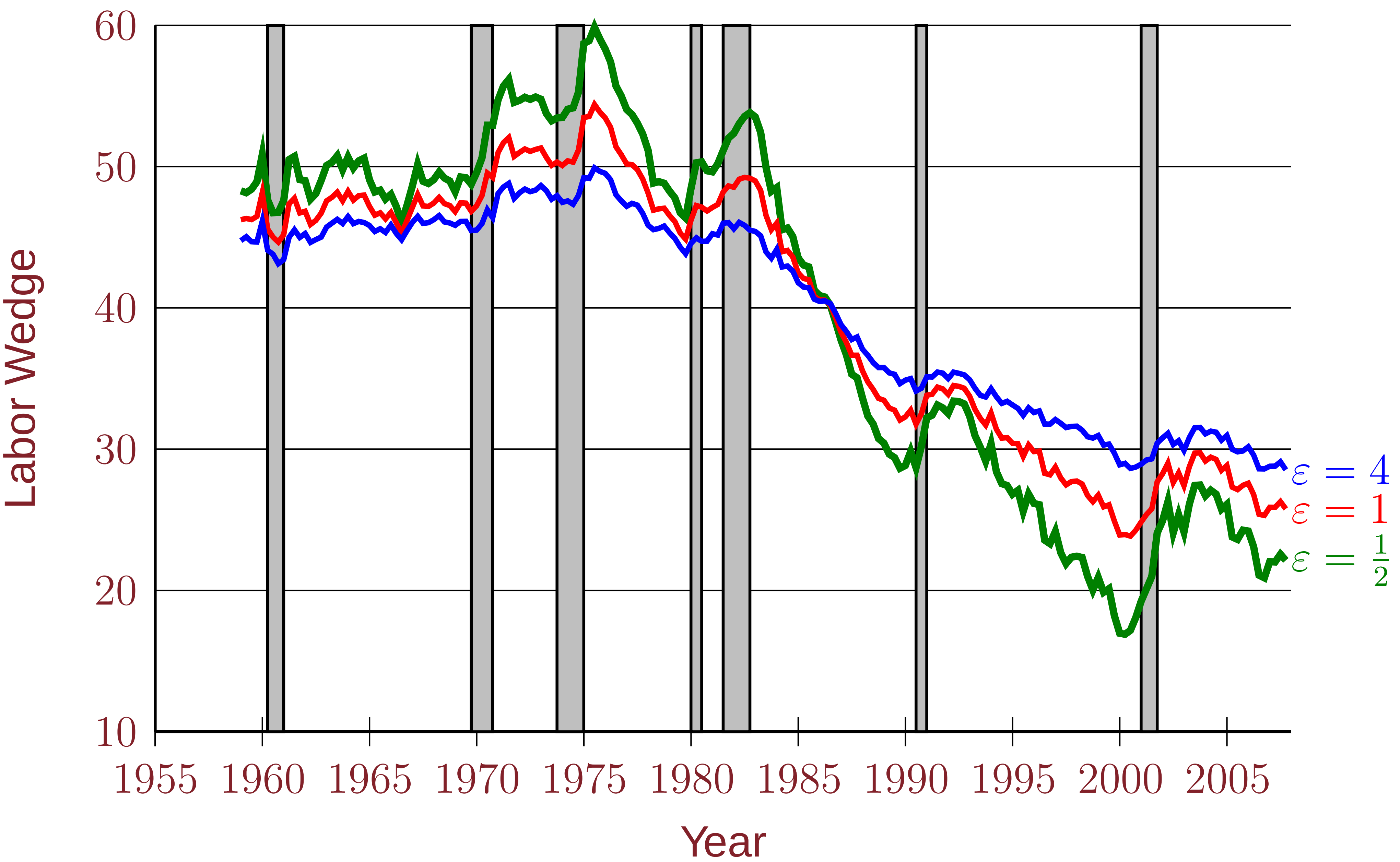
$\varepsilon = 1$
$\varepsilon = \frac{1}{2}$

Labor Wedge



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$\varepsilon = 4$
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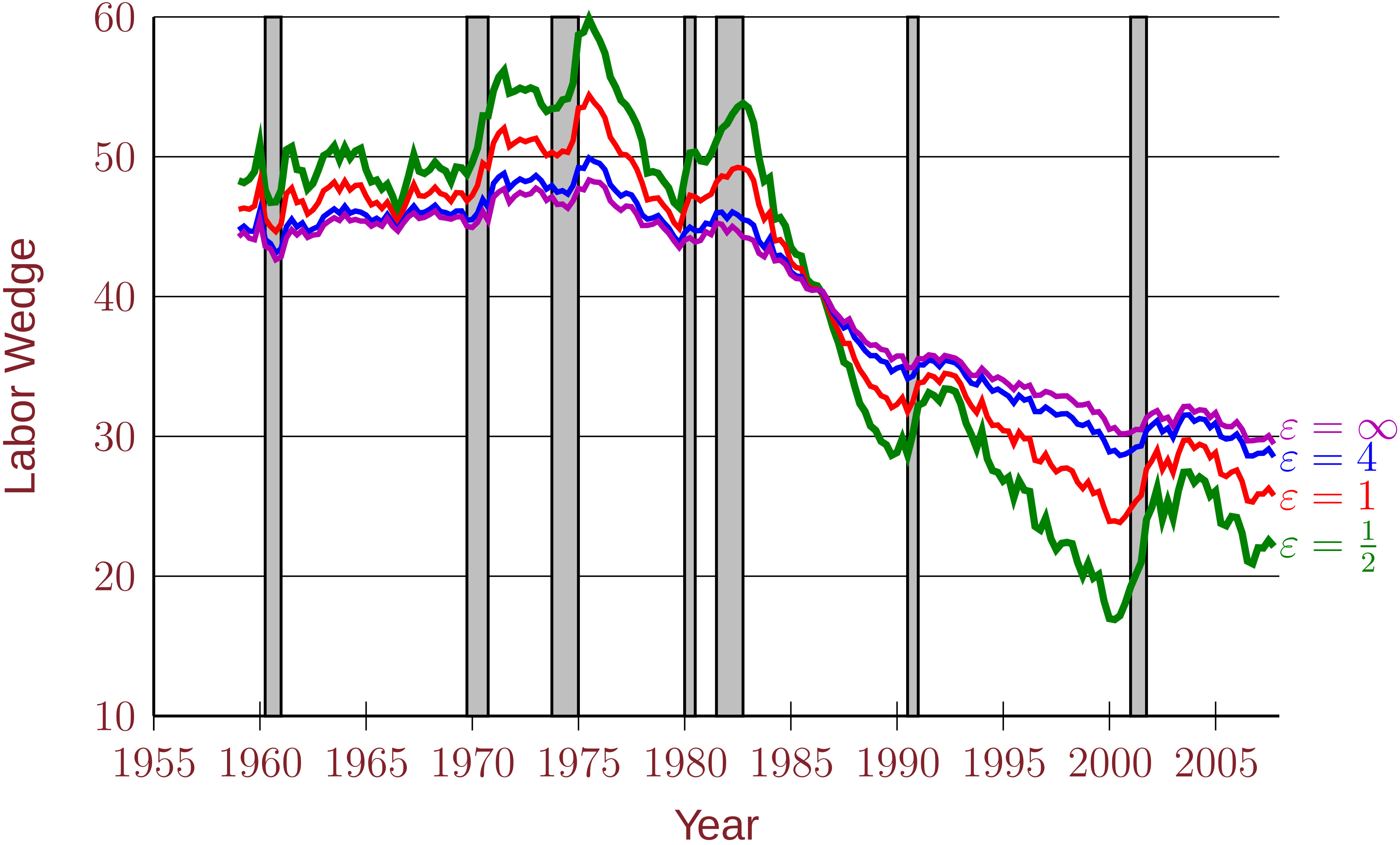
$\varepsilon = \frac{1}{2}$

Labor Wedge



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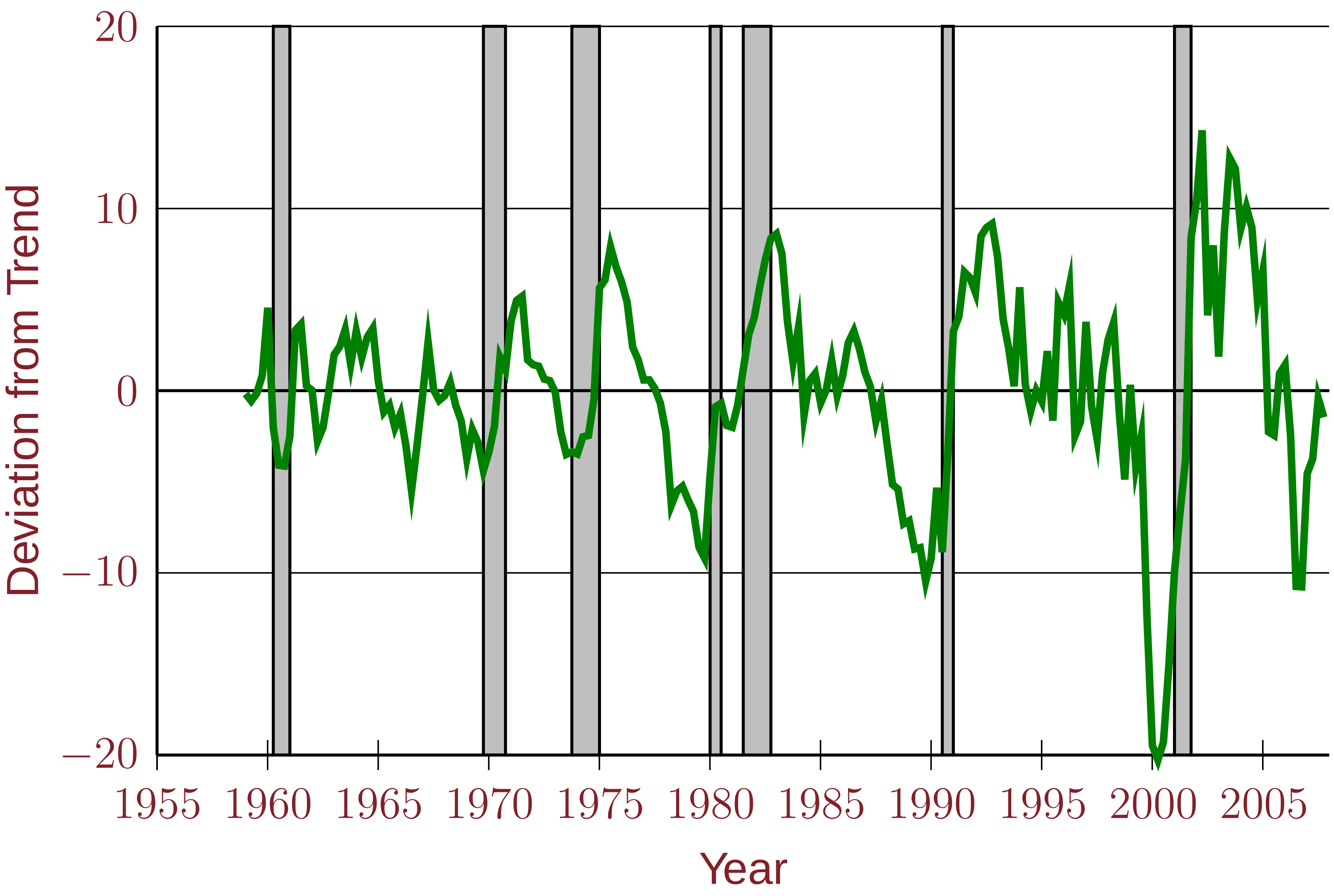
Labor Wedge



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Deviation from Trend
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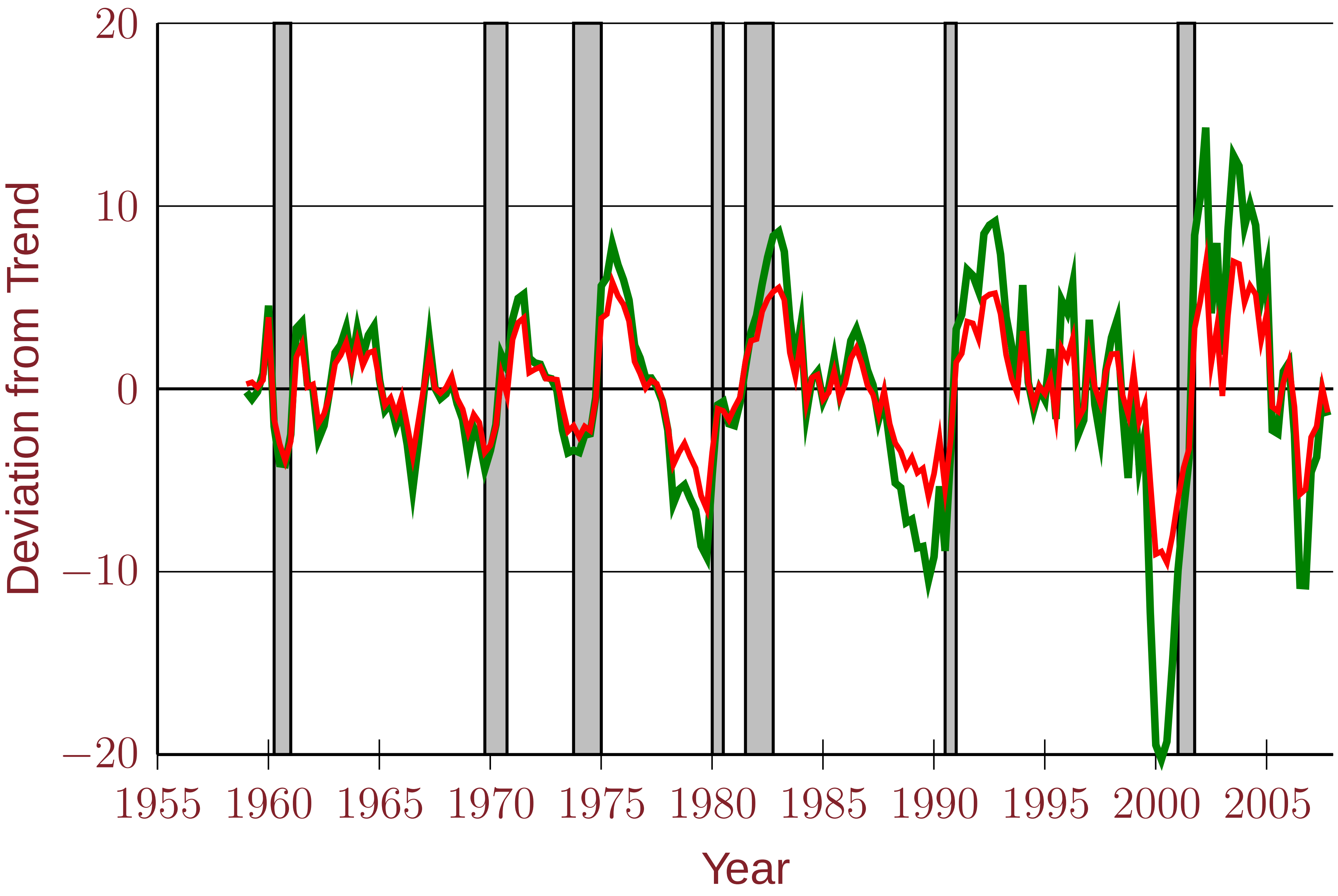
Labor Wedge



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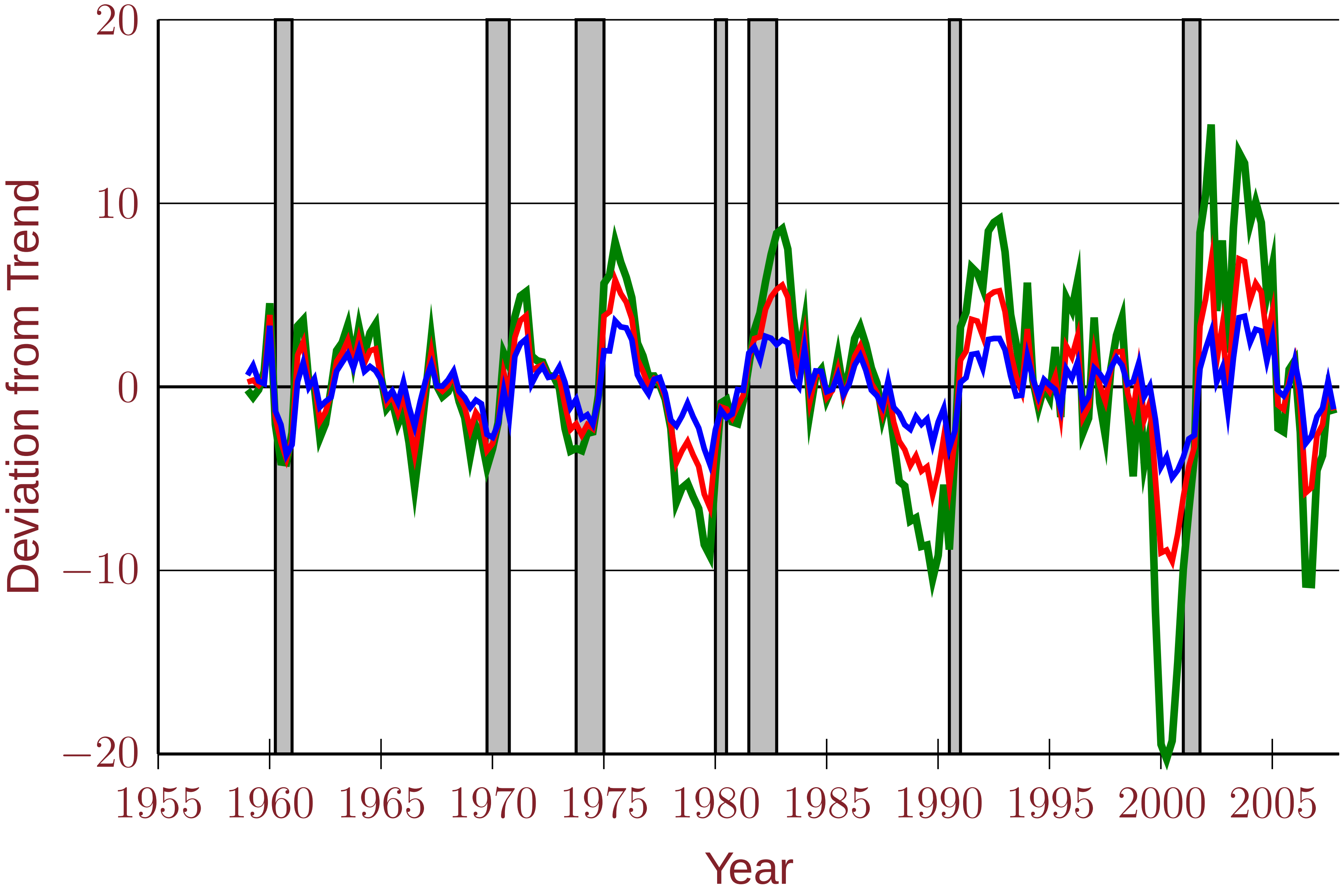

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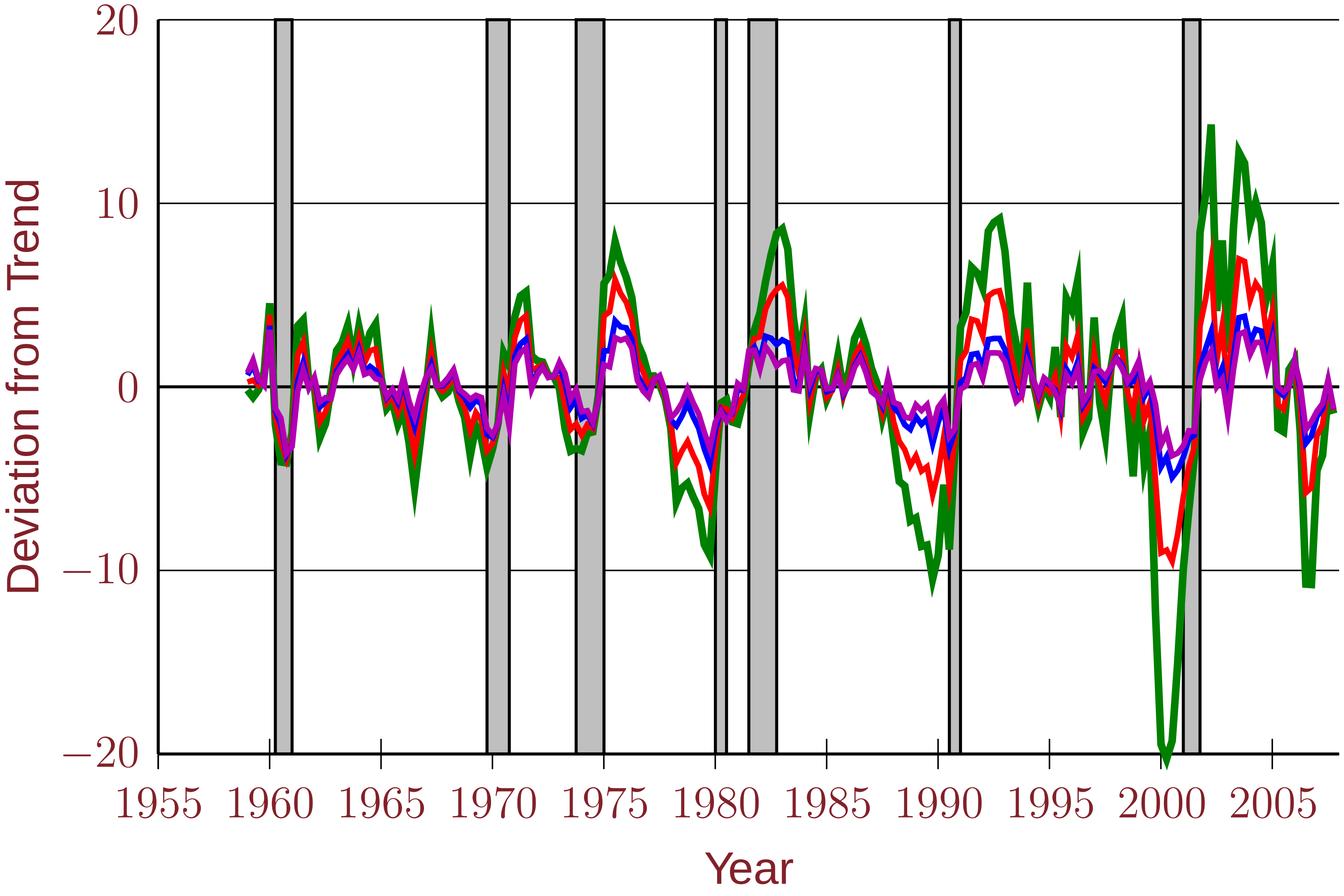

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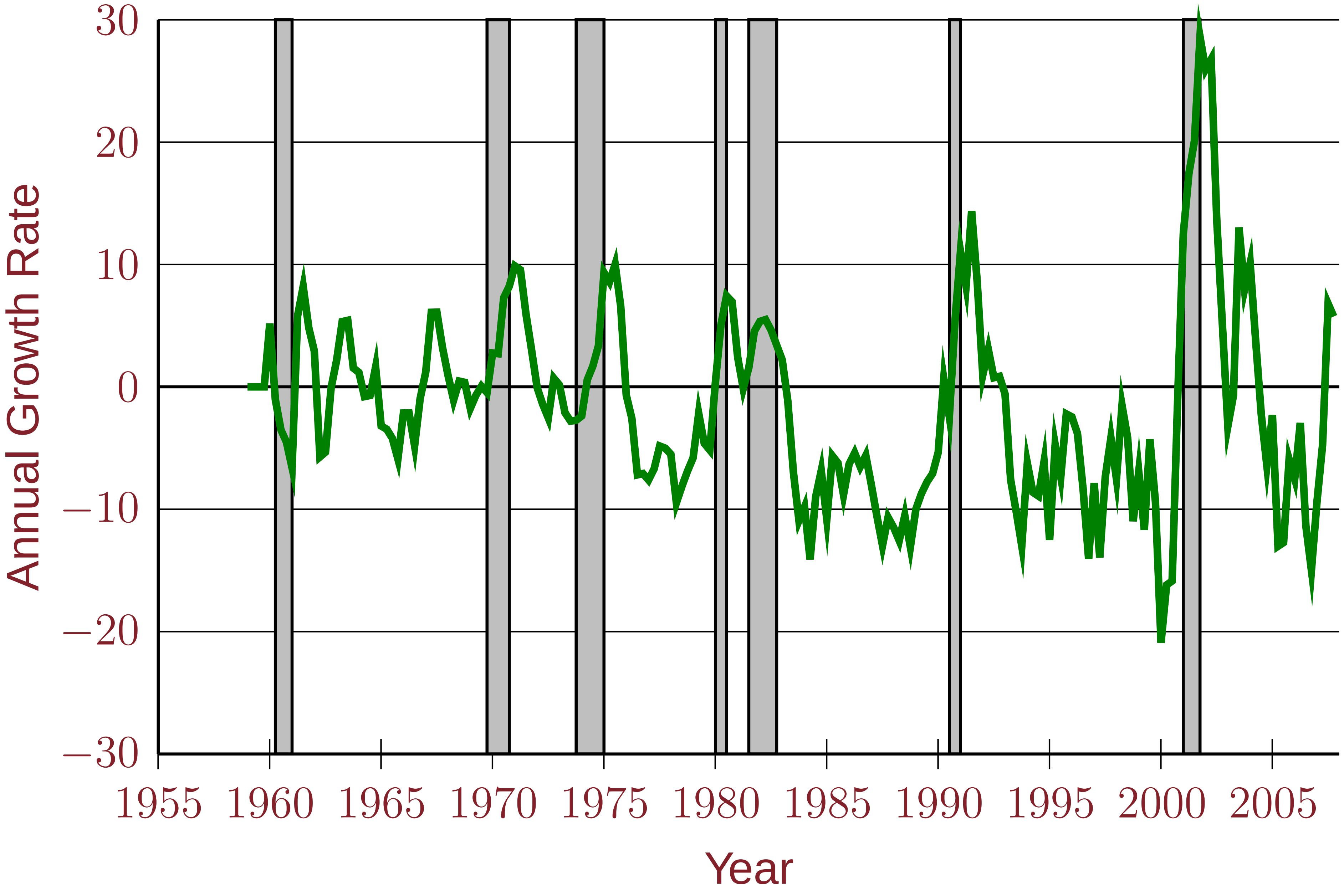

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Annual Growth Rate
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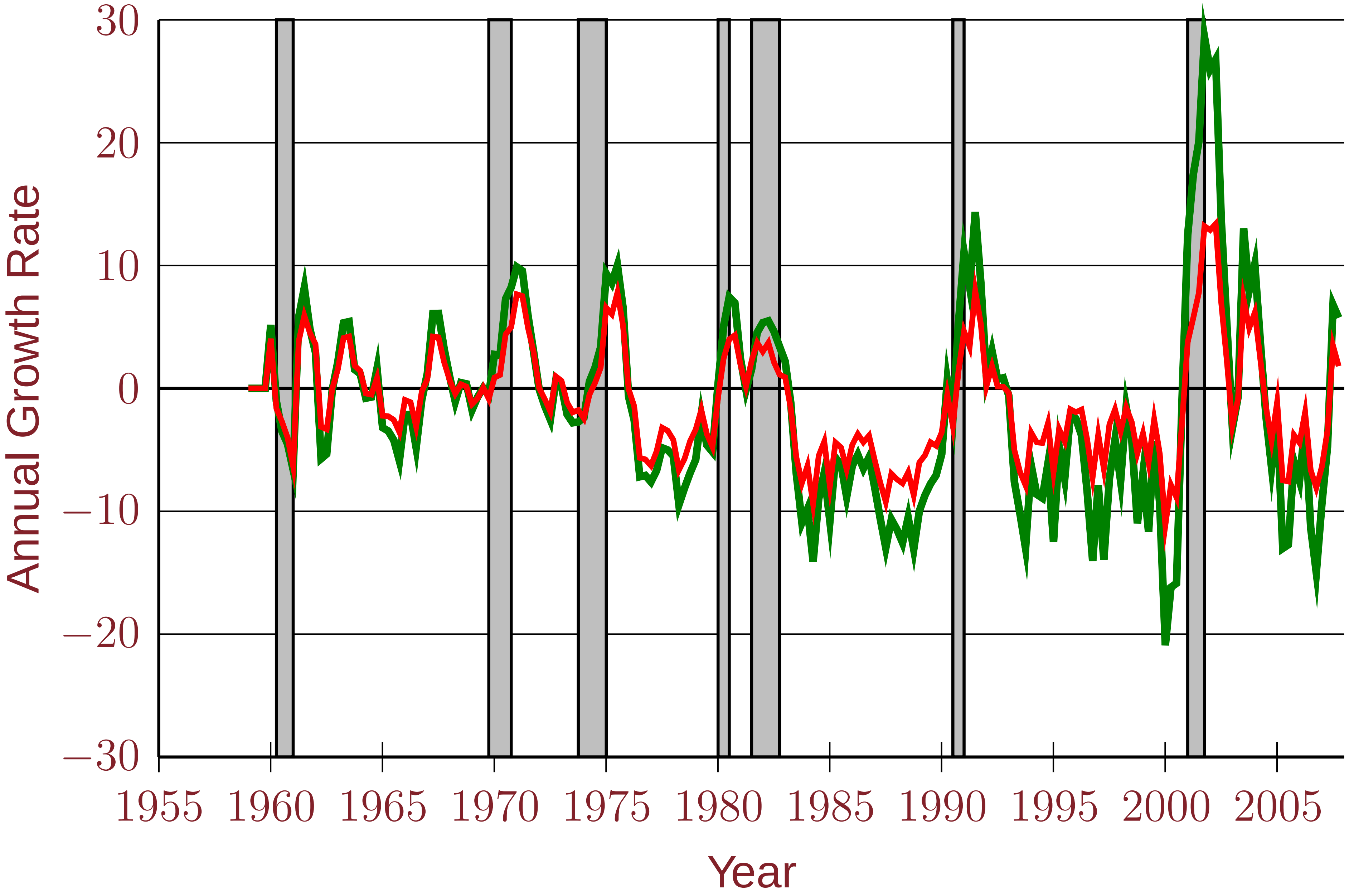
Labor Wedge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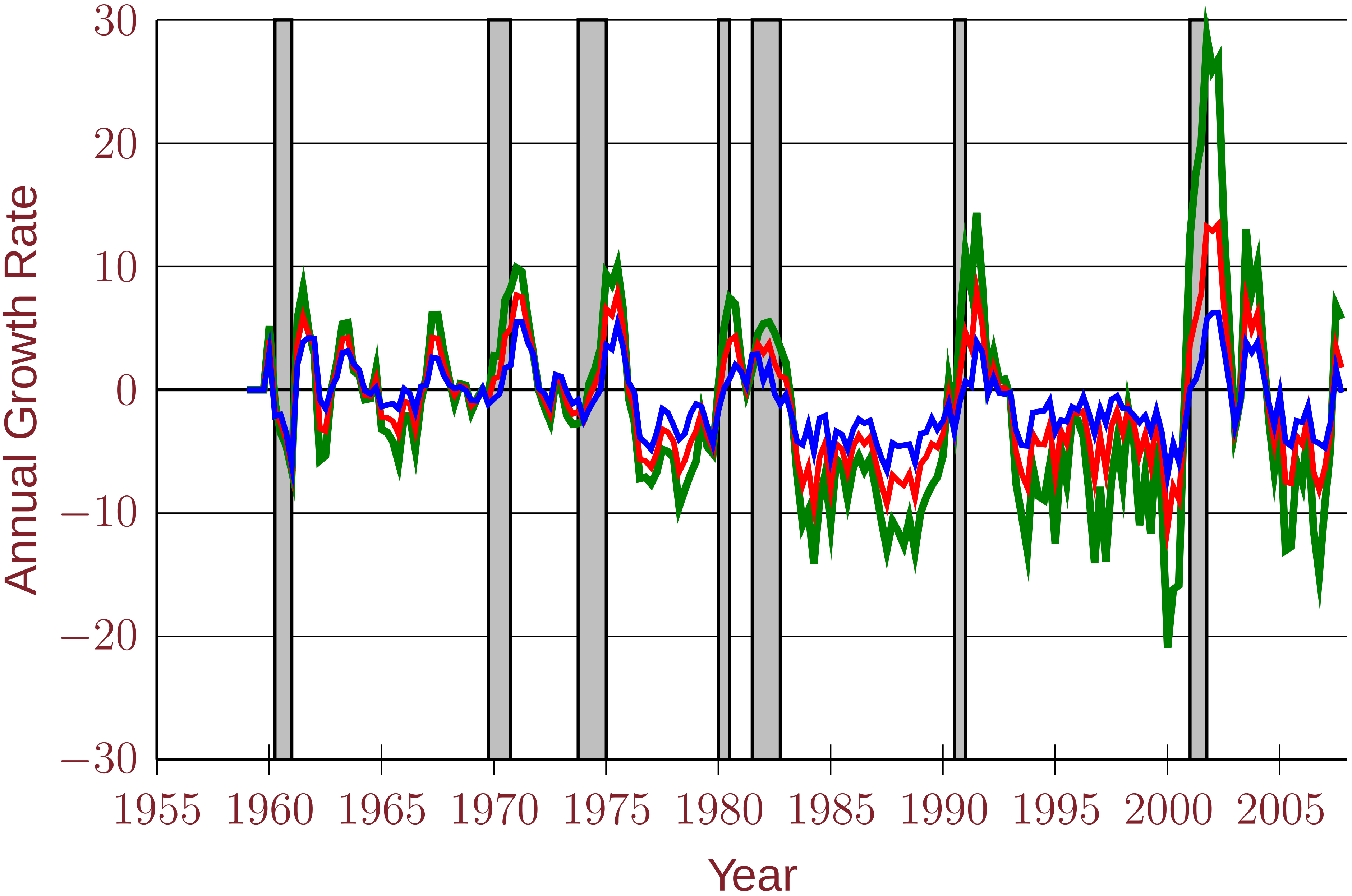

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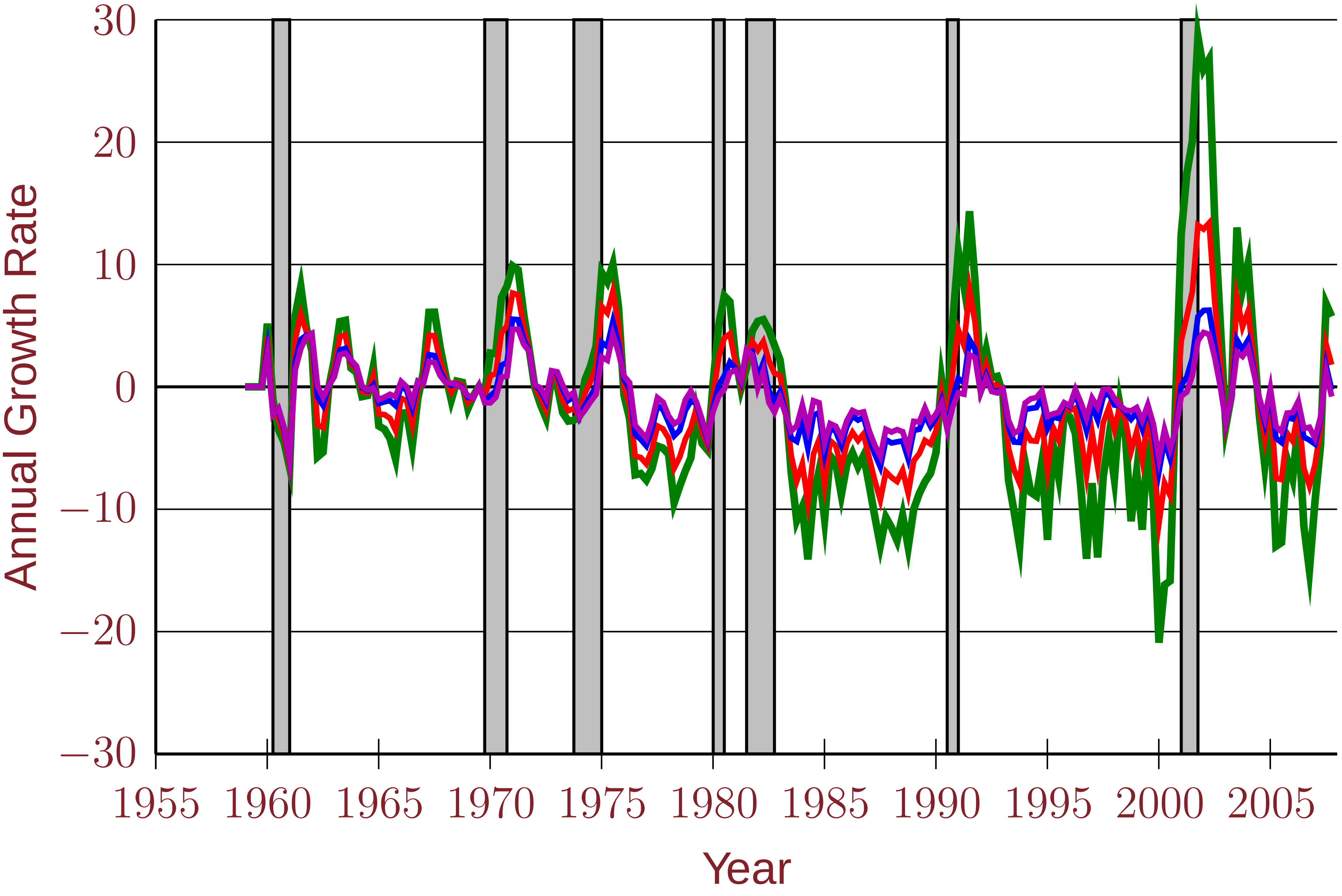

Annual Growth Rate
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Summary

detrended

		τ			
		$\varepsilon = 0.5$	$\varepsilon = 1$	$\varepsilon = 4$	$\varepsilon = \infty$
	s.d.	0.055	0.031	0.018	0.014
c/y	0.010	0.338	0.278	0.049	-0.131
h	0.013	-0.795	-0.835	-0.745	-0.628

annual growth rate

		τ			
		$\varepsilon = 0.5$	$\varepsilon = 1$	$\varepsilon = 4$	$\varepsilon = \infty$
	s.d.	0.079	0.045	0.027	0.022
c/y	0.015	0.256	0.163	-0.088	-0.260
h	0.018	-0.803	-0.835	-0.733	-0.617

Possible Resolutions

The Issue

- hours are 50 percent more volatile than consumption/output
- their correlation is -0.6

  $\Rightarrow$ the labor wedge is countercyclical

Possible Resolutions

- shocks to the marginal disutility of work γ

Possible Resolutions

- shocks to the marginal disutility of work γ

> "Sargent (1976) has attempted to remedy this fatal flaw by hypothesizing that the persistent and large fluctuations in unemployment reflect merely corresponding swings in the natural rate itself. In other words, what happened to the United States in the 1930's was a severe attack of contagious laziness! I can only say that, despite Sargent's ingenuity, neither I nor, I expect, most others at least of the nonmonetarists' persuasion are quite ready yet to turn over the field of economic fluctuations to the social psychologist"
> — Franco Modigliani (1977, p. 6)

Possible Resolutions

- shocks to the marginal disutility of work γ

"Alternatively, one could explain the observed pattern without a procyclical real wage by positing that tastes for consumption relative to leisure vary over time. Recessions are then periods of 'chronic laziness.' As far as I know, no one has seriously proposed this explanation of the business cycle" — N. Gregory Mankiw (1989, p. 82)

Possible Resolutions

- shocks to the marginal disutility of work γ
  - Hall (1987)
  - Rotemberg and Woodford (1997)
  - Erceg, Henderson, and Levin (2000)
  - Smets and Wouters (2003)
  - Galí and Rabanal (2004)

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- shocks to the wage markup
  - Smets and Wouters (2003) and (2007)

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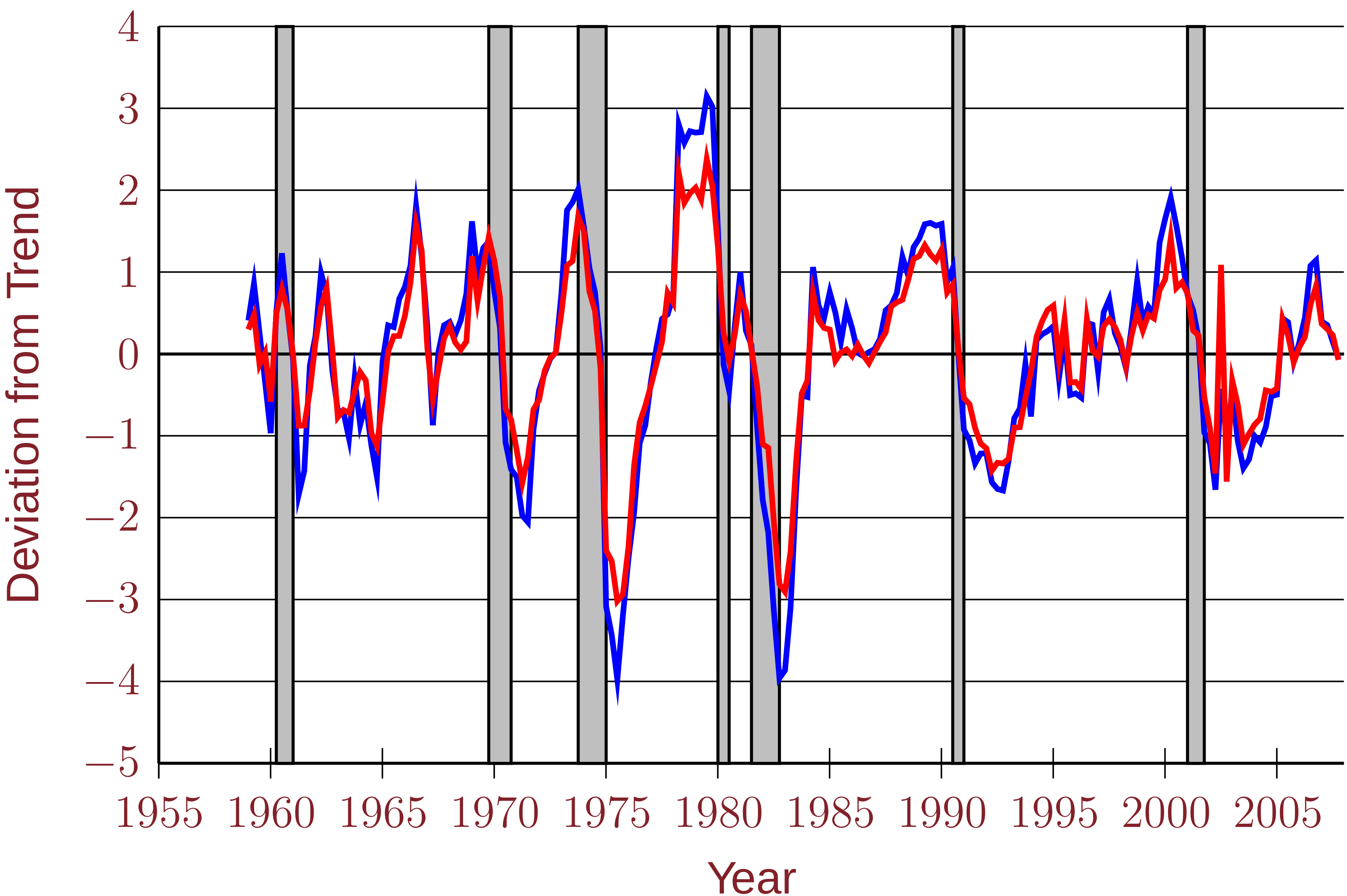
- nominal wage rigidities
- job search

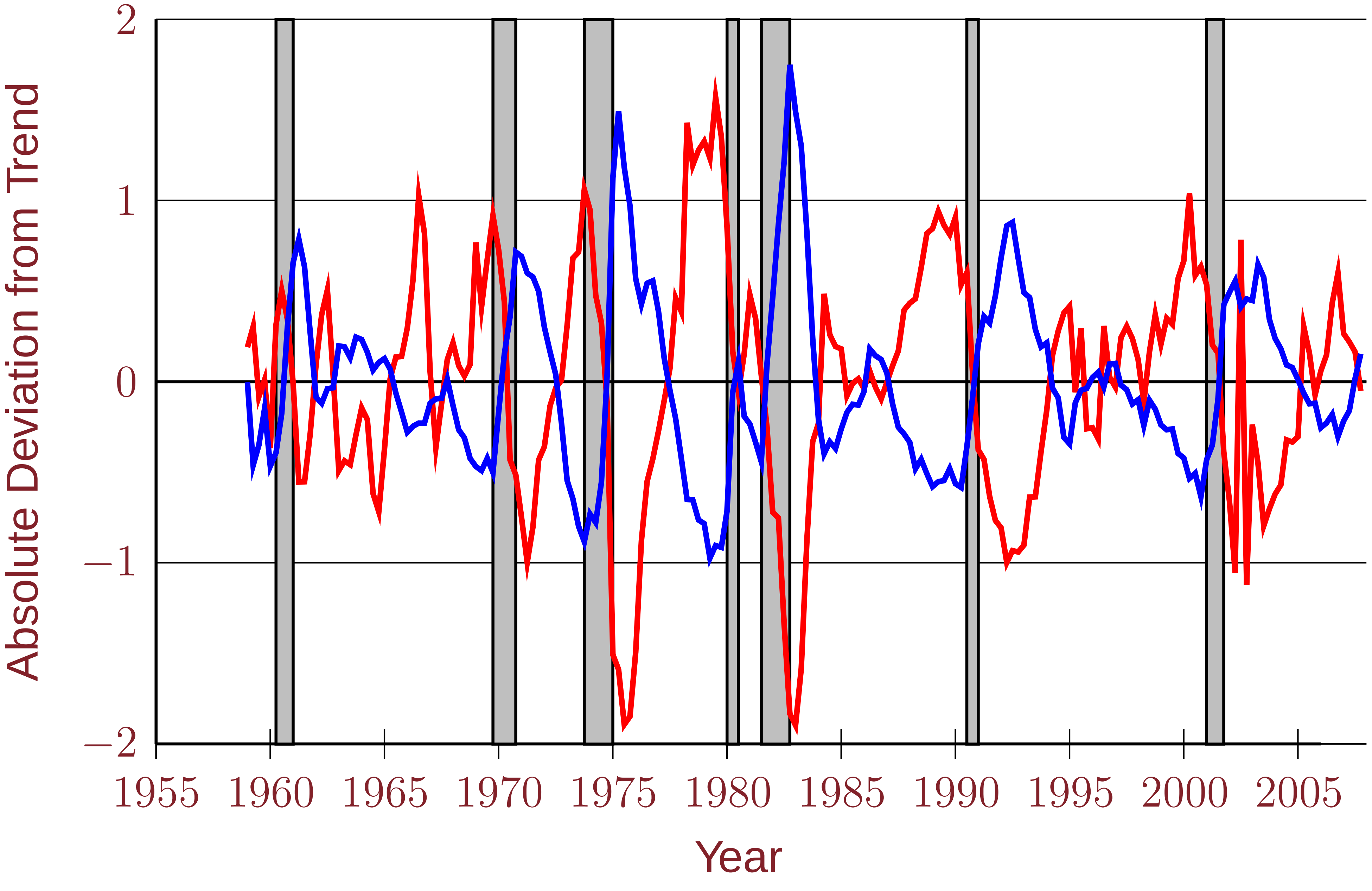
Why Search?

- model structure is reasonable for business cycles
  - focus on employment rather than hours
  - focus on unemployment rather than nonemployment

Employment versus Hours



Unemployment versus Employment



Why Search?

- model structure is reasonable for business cycles
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- plausible environment for analyzing wage rigidities
  - Barro (1977) critique of implicit contract models

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- model structure is reasonable for business cycles
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- plausible environment for analyzing wage rigidities
  - Barro (1977) critique of implicit contract models
- caveat: search frictions naturally reduce volatility in employment

Baseline Search Model

Baseline Search Model

- representative household
  - consumes
  - supplies labor
  - owns firms
- representative firm
  - produces consumption good using labor
  - recruits workers using labor
- government: constant tax, time-varying transfer, no debt
- wages are bargained by workers and firms

States and History

- time is $t = 0, 1, 2, \ldots$
- state of the economy at t is s_t
- history of the economy at t is $s^t \equiv \{s_0, s_1, \ldots, s_t\}$
  - productivity $z(s^t)$
- $\Pi(s^t)$ is time-0 probability of history s^t

Two Technologies

- divide $n(s^t) = \ell(s^t) + v(s^t)$ workers between two technologies
- constant-returns-to-scale production: $y(s^t) = z(s^t)\ell(s^t)$
- constant-returns-to-scale recruiting:

$$n(s^{t+1}) = (1-x)n(s^t) + v(s^t)\mu(\theta(s^t))$$

  - x: employment exit probability
  - θ: recruiter-to-unemployment ratio
  - $\mu(\theta)$: new hires per recruiter
    - continuous and nonincreasing on $(0, \infty)$
    - $\mu(0) = \infty$ and $\mu(\infty) = 0$
    - $f(\theta) \equiv \mu(\theta)\theta$ is nondecreasing

Firm Problem

- firm chooses $\{\nu(s^t)\}$, where $\nu(s^t) \equiv v(s^t)/n(s^t)$, to maximize

$$J(s^0, n_0) = \sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t) n(s^t) \big(z(s^t)(1 - \nu(s^t)) - w(s^t) \big)$$

$$\text{s.t. } n(s^{t+1}) = n(s^t) \big(1 - x + \nu(s^t) \mu(\theta(s^t)) \big)$$

  taking $n_0 = n(s^0)$ and $\{q_0(s^t), w(s^t), \theta(s^t)\}$ as given

  - $q_0(s^t)$: price of an Arrow-Debreu security
  - $w(s^t)$: wage in history s^t (in units of history-s^t consumption)

- firm's value is linear in n_0, $J(s^0, n_0) = \bar{J}(s^0) n_0$

Individual Preferences

- representative household with many individual members $i \in [0, 1]$
  - i has time-separable preferences over consumption and leisure
  - felicity $\log c_i - \gamma$ if employed and consuming c_i
  - felicity $\log c_i$ if unemployed and consuming c_i
- household maximizes the sum of its members' utility
  - $c_i(s^t)$ is the same for all i
- standard trick for getting the complete markets allocation
  - Merz (1995)

Household Preferences

- household acts as if it has preferences

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left(\log c(s^t) - \gamma n(s^t) \right),$$

  - $c(s^t)$: consumption in history s^t
  - $n(s^t)$: employment rate in history s^t
  - $\beta \in (0, 1)$: discount factor
  - γ: disutility of working

Household Constraints

- single lifetime budget constraint

$$a_0 = \sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t)\Big(c(s^t) - (1-\tau)w(s^t)n(s^t) - T(s^t)\Big)$$

  - a_0: initial assets
  - τ: constant labor income tax rate
  - $T(s^t)$: lump-sum transfer in history s^t

- sequence of employment constraints:

$$n(s^{t+1}) = (1-x)n(s^t) + f(\theta(s^t))(1-n(s^t))$$

Household Problem

- household chooses $\{c(s^t)\}$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left(\log c(s^t) - \gamma n(s^t)\right)$$

s.t. $a_0 = \sum_{t=0}^{\infty} \sum_{s^t} q_0(s^t) \left(c(s^t) - (1-\tau) w(s^t) n(s^t) - T(s^t)\right)$

and $n(s^{t+1}) = (1-x) n(s^t) + f(\theta(s^t))(1 - n(s^t))$,

taking a_0, $n_0 = n(s^0)$, and $\{q_0(s^t), w(s^t), \theta(s^t), \tau, T(s^t)\}$ as given

Government

- balanced budget: $T(s^t) = \tau w(s^t) n(s^t)$
- note that Ricardian equivalence holds

Wage Bargaining

- define two objects:
  - $\tilde{J}_n(s^t, w)$: value of paying a worker w in s^t, $w(s^{t'})$ thereafter, compared to not employing the worker
  - $\tilde{V}_n(s^t, w)$: value of having a worker paid w in s^t, $w(s^{t'})$ thereafter, compared to having the worker unemployed
    - evaluated at the equilibrium level of assets and employment

- the wage satisfies the Nash bargaining solution:

$$w(s^t) = \arg\max_w \tilde{V}_n(s^t, w)^{\phi} \tilde{J}_n(s^t, w)^{1-\phi},$$

where ϕ is workers' bargaining power

Market Clearing

- goods market clearing: $c(s^t) = z(s^t)n(s^t)(1 - \nu(s^t))$ for all t
- definition of θ: $\theta(s^t) = \dfrac{\nu(s^t)n(s^t)}{1 - n(s^t)}$
- capital market clearing: $a(s^t) = J(s^t, n(s^t))$ for all t
  - $a(s^t)$: household assets in history s^t
  - $J(s^t, n(s^t))$: value of firm in history s^t
  - this is implied by the other equations

Equilibrium

- a_0, n_0, and $\{c(s^t), n(s^t), \nu(s^t), \theta(s^t), q_0(s^t), w(s^t), T(s^t)\}$ such that:
  - firm problem is solved
  - household problem is solved
  - government budget constraint is satisfied
  - wages satisfy the Nash bargaining solution
  - goods market clears
  - θ is the ratio of recruiters to unemployed

Firm's Problem

- firm's value is linear in employment, $J(s^t, n) = \bar{J}(s^t)n$

Firm's Problem

- firm's value is linear in employment, $J(s^t, n) = \bar{J}(s^t)n$
- express per-worker value recursively

$$\bar{J}(s^t) = \max_{\nu} \left(z(s^t)(1-\nu) - w(s^t) + \left(\nu\mu(\theta(s^t)) + 1 - x\right) \sum_{s^{t+1}|s^t} q_t(s^{t+1}) \bar{J}(s^{t+1}) \right)$$

Firm's Problem: Main Results

- first order condition for ν:

$$z(s^t) = \mu(\theta(s^t)) \sum_{s^{t+1}|s^t} q_t(s^{t+1}) \bar{J}(s^{t+1})$$

firms are indifferent between production and recruiting

- simplify Bellman equation:

$$\bar{J}(s^t) = z(s^t)\left(1 + \frac{1-x}{\mu(\theta(s^t))}\right) - w(s^t)$$

value of job is current output plus saved recruiting minus wage

- the value of paying a worker w is

$$\tilde{J}_n(s^t, w) = w(s^t) - w + \bar{J}(s^t)$$

Worker's Problem

- express worker's problem recursively

$$V(s^t, a, n) = \max_{\{a(s^{t+1})\}} \left(\log c - \gamma n + \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V\left(s^{t+1}, a(s^{t+1}), n'\right) \right),$$

where $c = a + (1-\tau)w(s^t)n + T(s^t) - \sum_{s^{t+1}|s^t} q_t(s^{t+1})a(s^{t+1})$

and $n' = (1-x)n + f(\theta(s^t))(1-n)$

- first order condition for future assets:

$$\frac{q_t(s^{t+1})}{\tilde{c}(s^t, a, n)} = \beta \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_a(s^{t+1}, a(s^{t+1}), n')$$

- envelope condition for current assets:

$$V_a(s^t, a, n) = 1/\tilde{c}(s^t, a, n)$$

Worker's Problem: Main Results

- intertemporal Euler equation

$$q_t(s^{t+1}) = \beta \frac{\Pi(s^{t+1})c(s^t)}{\Pi(s^t)c(s^{t+1})}$$

- envelope condition for current employment:

$$V_n(s^t, a(s^t), n(s^t)) = \frac{(1-\tau)w(s^t)}{c(s^t)} - \gamma$$
$$+ \beta(1 - x - f(\theta(s^t))) \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_n(s^{t+1}, a(s^{t+1}), n(s^{t+1}))$$

- value of having a worker paid w rather than unemployed is

$$\tilde{V}_n(s^t, w) = \frac{(1-\tau)(w - w(s^t))}{c(s^t)} + V_n(s^t, a(s^t), n(s^t))$$

Wage Setting: Main Result

- wage solves $w(s^t) = \arg\max_w \tilde{V}_n(s^t, w)^{\phi} \tilde{J}_n(s^t, w)^{1-\phi}$
- from previous expressions, this implies implies

$$\frac{(1-\tau)w(s^t)}{c(s^t)} = \phi \frac{(1-\tau)z(s^t)(1+\theta(s^t))}{c(s^t)} + (1-\phi)\gamma$$

- after-tax wage (in utils) is weighted average of
  - after-tax output (in utils) produced by
    1. the worker, $(1-\tau)z(s^t)$
    2. other workers freed from recruiting, $(1-\tau)z(s^t)\theta(s^t)$
  - marginal rate of substitution between consumption and leisure

Equilibrium

- consumption, wage, and firm value are proportional to z
  - $c(s^t) = \bar{c}z(s^t)$
  - $w(s^t) = \bar{w}z(s^t)$
  - $\bar{J}(s^t) = \bar{J}z(s^t)$
- recruiters/unemployed, employment, and worker value are constant:
  - $\theta(s^t) = \bar{\theta}$
  - $n(s^t) = \bar{n}$
  - $V_n(s^t, a(s^t), n(s^t)) = \bar{V}_n$
- constant measured labor wedge $\hat{\tau}$
- note that this equilibrium requires $n_0 = \dfrac{f(\bar{\theta})}{f(\bar{\theta}) + x}$

Explanation

- productivity shock does not effect the efficiency of recruiting
- income and substitution effects offset

Planner's Problem

- planner chooses $\{\nu(s^t), \theta(s^t)\}$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \Big(\log \left(z(s^t) n(s^t)(1-\nu(s^t)) \right) - \gamma n(s^t) \Big)$$

s.t. $n(s^{t+1}) = (1-x)n(s^t) + f(\theta(s^t))(1-n(s^t))$

and $\theta(s^t) = \dfrac{\nu(s^t)n(s^t)}{1-n(s^t)}$

- productivity terms $z(s^t)$ are additively separable
- the planner's solution coincides with equilibrium if
  - $\tau = 0$: no distortionary taxes
  - $\phi = 1 - \dfrac{\theta f'(\theta)}{f(\theta)}$: Mortensen-Hosios condition holds

Extensions

Labor Force Participation

- household chooses $\{c(s^t), u(s^t), n(s^{t+1})\}$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \Big(\log c(s^t) - \gamma_n n(s^t) - \gamma_u u(s^t) \Big)$$

  subject to $n(s^{t+1}) = (1-x)n(s^t) + f(\theta(s^t))u(s^t)$ and budget constraint

- express household problem recursively:

$$V(s^t, a, n) = \max_{\{a(s^{t+1})\}, u \in [0, 1-n]} \Bigg(\log c - \gamma_n n - \gamma_u u \\ + \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V\Big(s^{t+1}, a(s^{t+1}), n'\Big) \Bigg)$$

  where $c = a + (1-\tau)w(s^t)n + T(s^t) - \sum_{s^{t+1}|s^t} q_t(s^{t+1})a(s^{t+1})$

  and $n' = (1-x)n + f(\theta(s^t))u$

Labor Force Participation

- Euler equation, marginal value of employment unchanged
- first order condition for u:

$$\gamma_u = \beta f(\theta(s^t)) \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} \left(\frac{(1-\tau)w(s^{t+1})}{c(s^{t+1})} - \gamma_n + \frac{\gamma_u(1-x)}{f(\theta(s^{t+1}))} \right)$$

Labor Force Participation

- Euler equation, marginal value of employment unchanged
- first order condition for u:

$$\gamma_u = \beta f(\theta(s^t)) \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} \left(\frac{(1-\tau)w(s^{t+1})}{c(s^{t+1})} - \gamma_n + \frac{\gamma_u(1-x)}{f(\theta(s^{t+1}))} \right)$$

- firm's problem is unchanged
- wage is unchanged
- constant equilibrium employment, unemployment, and labor wedge

Variable Hours

- household chooses $\{c(s^t)\}$ to maximize

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\Pi(s^t)\left(\log c(s^t)-\frac{\gamma\varepsilon}{1+\varepsilon}n(s^t)h(s^t)^{\frac{1+\varepsilon}{\varepsilon}}\right)$$

  subject to the evolution of n and a budget constraint

- firm chooses $\{\nu(s^t)\}$ to maximize

$$J(s^0,n_0)=\sum_{t=0}^{\infty}\sum_{s^t}q_0(s^t)n(s^t)h(s^t)\big(z(s^t)(1-\nu(s^t))-w(s^t)\big)$$

$$\text{s.t. } n(s^{t+1})=n(s^t)\big(1-x+h(s^t)\nu(s^t)\mu(\theta(s^t))\big)$$

Variable Hours: Household Problem

- marginal value of employed worker:

$$V_n(s^t, a(s^t), n(s^t)) = \frac{(1-\tau)w(s^t)h(s^t)}{c(s^t)} - \frac{\gamma\varepsilon}{1+\varepsilon}h(s^t)^{\frac{1+\varepsilon}{\varepsilon}}$$
$$+ \beta(1 - x - f(\theta(s^t))) \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_n(s^{t+1}, a(s^{t+1}), n(s^{t+1}))$$

- value of worker paid w and working h instead of unemployed:

$$\tilde{V}_n(s^t, w, h) = \frac{(1-\tau)\big(wh - w(s^t)h(s^t)\big)}{c(s^t)}$$
$$- \frac{\gamma\varepsilon}{1+\varepsilon}\left(h^{\frac{1+\varepsilon}{\varepsilon}} - h(s^t)^{\frac{1+\varepsilon}{\varepsilon}}\right) + V_n(s^t, a(s^t), n(s^t))$$

Variable Hours: Firm Problem

- firm indifferent between recruiting and producing:

$$z(s^t) = \mu(\theta(s^t)) \sum_{s^{t+1}|s^t} q_t(s^{t+1}) \bar{J}(s^{t+1})$$

- value of job is current output plus saved recruiting minus wage

$$\bar{J}(s^t) = z(s^t) \frac{1 - x + h(s^t)\mu(\theta(s^t))}{\mu(\theta(s^t))} - w(s^t)h(s^t)$$

- value of worker paid w and working h

$$\tilde{J}_n(s^t, w, h) = (z(s^t) - w)h - (z(s^t) - w(s^t))h(s^t) + \bar{J}(s^t).$$

Variable Hours: Closing Model

- $(w(s^t), h(s^t)) = \arg\max_{w,h} \tilde{V}_n(s^t, w, h)^{\phi} \tilde{J}_n(s^t, w, h)^{1-\phi}$
  - $h(s^t)$ set to maximize joint surplus

$$h(s^t) = \left(\frac{(1-\tau)z(s^t)}{\gamma c(s^t)}\right)^{\varepsilon}.$$

  - $w(s^t)$ set to divide the surplus

$$w(s^t) = \left(\phi\left(1 + \frac{\theta(s^t)}{h(s^t)}\right) + (1-\phi)\frac{\varepsilon}{1+\varepsilon}\right) z(s^t)$$

- constant equilibrium employment, hours, and labor wedge

Unemployment Benefits

- unemployed workers get an after-tax benefit $Bw(s^t)$
  - tied to current wage, not past wage
- government budget constraint: $T(s^t) = \tau w(s^t)n(s^t) - Bw(s^t)(1-n(s^t))$
- constant equilibrium employment and labor wedge (prove it)

Government Spending

- government spends $g(s^t)$
- preferences $\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \big(\log c(s^t) - \gamma n(s^t) + \psi(g(s^t)) \big)$
- government budget constraint $T(s^t) + g(s^t) = \tau w(s^t) n(s^t)$
- resource constraint $c(s^t) + g(s^t) = z(s^t) n(s^t)(1 - \nu(s^t))$
- if $g(s^t) = \bar{g} z(s^t)$, constant employment and labor wedge (prove it)
- is this reasonable?
    - balanced budget requirement
    - optimal if $\psi(g) \equiv \log(g)$

Capital

Firm Problem

- firm chooses $\{\nu(s^t), n(s^{t+1}), k(s^{t+1})\}$ to maximize

$$\sum_{t=0}^{\infty}\sum_{s^t} q_0(s^t)\Big(z(s^t)k(s^t)^{\alpha}\big(n(s^t)(1-\nu(s^t))\big)^{1-\alpha} + (1-\delta)k(s^t) - k(s^{t+1}) - w(s^t)n(s^t)\Big),$$

  where firm growth satisfies

$$n(s^{t+1}) = n(s^t)\big(1 - x + \nu(s^t)\mu(\theta(s^t))\big),$$

  taking $k_0 = k(s^0)$, $n_0 = n(s^0)$, and $\{q_0(s^t), w(s^t), \theta(s^t)\}$ as given

- call the value of the firm $J(s^0, n_0, k_0)$
  - homogeneous of degree 1 in (n_0, k_0)

Firm's Problem: Recursive

- express the firm's problem recursively:

$$J(s^t, n, k) = \max_{\nu, k'} \left(z(s^t) k^{\alpha} \big(n(1-\nu)\big)^{1-\alpha} + (1-\delta)k - k' - nw(s^t) \right.$$
$$\left. + \sum_{s^{t+1}|s^t} q_t(s^{t+1}) J\Big(s^{t+1}, n\big(\nu\mu(\theta(s^t)) + 1 - x\big), k'\Big) \right)$$

Firm's Problem: Main Results

- marginal value of worker is her and recruiters' output minus wage:

$$J_n(s^t, n(s^t), k(s^t)) \\ = (1-\alpha) z(s^t) \left(\frac{k(s^t)}{n(s^t)(1-\nu(s^t))} \right)^{\alpha} \left(1 + \frac{1-x}{\mu(\theta(s^t))} \right) - w(s^t)$$

- firms are indifferent between production and recruiting:

$$(1-\alpha) z(s^t) \left(\frac{k(s^t)}{n(s^t)(1-\nu(s^t))} \right)^{\alpha} \\ = \mu(\theta(s^t)) \sum_{s^{t+1}|s^t} q_t(s^{t+1}) J_n(s^{t+1}, n(s^{t+1}), k(s^{t+1}))$$

- firms are indifferent about purchasing capital:

$$1 = \sum_{s^{t+1}|s^t} q_t(s^{t+1}) \left(\alpha z(s^{t+1}) \left(\frac{k(s^{t+1})}{n(s^{t+1})(1-\nu(s^{t+1}))} \right)^{\alpha-1} + 1 - \delta \right)$$

Worker's Problem

- intertemporal Euler equation

$$q_t(s^{t+1}) = \beta \frac{\Pi(s^{t+1})c(s^t)}{\Pi(s^t)c(s^{t+1})}$$

- marginal value of an employed worker (in utils) is

$$V_n(s^t, a(s^t), n(s^t)) = \frac{(1-\tau)w(s^t)}{c(s^t)} - \gamma$$
$$+ \beta(1 - x - f(\theta(s^t))) \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_n(s^{t+1}, a(s^{t+1}), n(s^{t+1}))$$

Wage Setting: Main Result

- Nash bargaining solution implies

$$(1-\tau)w(s^t) = \phi \frac{(1-\tau)(1-\alpha)z(s^t)k(s^t)^\alpha}{\big(n(s^t)(1-\nu(s^t))\big)^\alpha}(1+\theta(s^t)) + (1-\phi)\gamma c(s^t)$$

- after-tax wage is weighted average of
  - after-tax marginal product of labor produced by
    1. the worker and
    2. the $\theta(s^t)$ other workers freed from recruiting
  - marginal rate of substitution between consumption and leisure

Market Clearing

- goods market clearing:

$$c(s^t) + k(s^{t+1}) = z(s^t)k(s^t)^\alpha \Big(n(s^t)(1-\nu(s^t))\Big)^{1-\alpha} + (1-\delta)k(s^t)$$

- law of motion for employment:

$$n(s^{t+1}) = (1-x)n(s^t) + f(\theta(s^t))(1-n(s^t))$$

- definition of θ: $\theta(s^t) = \dfrac{\nu(s^t)n(s^t)}{1-n(s^t)}$

Balanced Growth

- suppose $\log z(s^{t+1}) = \log z(s^t) + \bar{s}$
- consumption, capital, and wage grow at rate $\bar{s}/(1-\alpha)$
  - $c(s^t) = \bar{c} z(s^t)^{\frac{1}{1-\alpha}}$
  - $k(s^t) = \bar{k} z(s^t)^{\frac{1}{1-\alpha}}$
  - $w(s^t) = \bar{w} z(s^t)^{\frac{1}{1-\alpha}}$
- recruiters/unemployed and employment are constant:
  - $\theta(s^t) = \bar{\theta}$
  - $n(s^t) = \bar{n}$

Deterministic Trend

- suppose $\log z(s^t) = \bar{s}t + s_t$, where s_t is persistent
  - first order Markov process, transition matrix $\pi(s_{t+1}|s_t)$
- define relative consumption, capital, and wage:
  - $c(s^t) = \tilde{c}(s^t)e^{\frac{\bar{s}t}{1-\alpha}}$
  - $k(s^t) = \tilde{k}(s^t)e^{\frac{\bar{s}t}{1-\alpha}}$
  - $w(s^t) = \tilde{w}(s^t)e^{\frac{\bar{s}t}{1-\alpha}}$

  these three variables are stationary
- so are recruiters/unemployed $\theta(s^t)$ and employment $n(s^t)$

Stationary Version of Key Equations

- firms indifferent about purchasing capital

$$e^{\frac{\bar{s}}{1-\alpha}} = \beta \sum_{s^{t+1}|s^t} \pi(s_{t+1}|s_t) \frac{\tilde{c}(s^t)}{\tilde{c}(s^{t+1})} \left(\alpha e^{s_{t+1}} \left(\frac{\tilde{k}(s^{t+1})}{n(s^{t+1})(1-\nu(s^{t+1}))} \right)^{\alpha-1} + 1 - \delta \right)$$

- interior condition for recruiting, wage equation:

$$(1-\alpha)e^{s_t} \left(\frac{\tilde{k}(s^t)}{n(s^t)(1-\nu(s^t))} \right)^{\alpha} =$$

$$\beta\mu(\theta(s^t)) \sum_{s^{t+1}|s^t} \pi(s_{t+1}|s_t) \frac{\tilde{c}(s^t)}{\tilde{c}(s^{t+1})} \left(-\frac{(1-\phi)\gamma\tilde{c}(s^{t+1})}{1-\tau} \right.$$

$$\left. +(1-\alpha)e^{s_{t+1}} \left(\frac{\tilde{k}(s^{t+1})}{n(s^{t+1})(1-\nu(s^{t+1}))} \right)^{\alpha} \left(\frac{1-x}{\mu(\theta(s^{t+1}))} + 1 - \phi - \phi\theta(s^{t+1}) \right) \right)$$

Stationary Version of Key Equations

- resource constraint:

$$\tilde{k}(s^{t+1})e^{\frac{\bar{s}}{1-\alpha}} = e^{s_t}\tilde{k}(s^t)^{\alpha}\left(n(s^t)(1-\nu(s^t))\right)^{1-\alpha} + (1-\delta)\tilde{k}(s^t) - \tilde{c}(s^t)$$

- unemployment rate:

$$n(s^{t+1}) = (1-x)n(s^t) + f(\theta(s^t))(1-n(s^t))$$

- relationship between ν and θ:

$$\theta(s^t) = \frac{\nu(s^t)n(s^t)}{1-n(s^t)}$$

  use this to eliminate ν from previous equations

- log linearize around steady state $s = 0$, $n = \bar{n}$, and $\tilde{k} = \bar{k}$

Log-Linearization

- posit

$$\Theta(s,n,\tilde{k}) \equiv \log\bar{\theta} + \theta_s s + \theta_n(\log n - \log\bar{n}) + \theta_k(\log\tilde{k} - \log\bar{k})$$
$$C(s,n,\tilde{k}) \equiv \log\bar{c} + c_s s + c_n(\log n - \log\bar{n}) + c_k(\log\tilde{k} - \log\bar{k})$$

  - eliminate $\theta(s^t)$ and $c(s^t)$ using these approximations
  - eliminate $n(s^{t+1})$ and $\tilde{k}(s^{t+1})$ using their laws of motion

- reduces to an equation of the form $T(s,n,\tilde{k}) = 0$
- impose $T(0,\bar{n},\bar{k}) = T_s(0,\bar{n},\bar{k}) = T_n(0,\bar{n},\bar{k}) = T_k(0,\bar{n},\bar{k}) = 0$
- solve for the unknown constants

Calibration

- discount factor $\beta = 0.996$
- employment exit probability $x = 0.034$
- average productivity growth $\bar{s} = 0.0012$
- productivity shocks $s_{t+1} = 0.98 s_t + 0.005 v_{t+1}$
- capital share $\alpha = 0.33$
- depreciation rate $\delta = 0.0028$: $k/y = 3.2$ in stochastic steady state
- tax rate $\tau = 0.4$
- bargaining power $\phi = 0.5$
- matching function $f(\theta) = 2.32\theta^{1/2}$
- disutility $\gamma = 0.471$: 5% unemployment rate in stochastic steady state

Log-Linearized System

- policy functions

$$\log\theta = \log 0.078 + 7.387s - 0.480\log(n/0.95) - 2.779\log(\tilde{k}/218.2),$$
$$\log\tilde{c} = \log 4.696 + 0.250s + 0.014\log(n/0.95) + 0.603\log(\tilde{k}/218.2)$$

- state equations

$$\log n_{+1} = \log 0.95 + 0.126s + 0.312\log(n/0.95) - 0.047\log(\tilde{k}/218.2),$$
$$\log\tilde{k}_{+1} = \log 218.2 + 0.020s + 0.019\log(n/0.95) + 0.991\log(\tilde{k}/218.2)$$

- write state as $m \equiv \{s, \log(n/\bar{n}), \log(\tilde{k}/\bar{k})\}$, so $m_+ = Am + Dv_+$
- local stability iff eigenvalues of A lie in unit circle
  - here they are 0.99, 0.98, and 0.31

Variance-Covariance

- using $m_+ = Am + Dv_+$, variance-covariance matrix is

$$\Sigma = \mathbb{E}\big(m_+ m_+'\big) = \mathbb{E}\big((Am + Dv_+)(m'A' + v_+' D')\big) = A\Sigma A' + DD'$$

- here

$$\Sigma = \begin{pmatrix} 25.253 & 3.175 & 19.505 \\ 3.175 & 0.560 & 0.469 \\ 19.505 & 0.469 & 46.550 \end{pmatrix} \varsigma^2.$$

- standard deviation of employment is $\varsigma\sqrt{0.560} = 0.004$

Variance-Covariance

- using $m_+ = Am + Dv_+$, variance-covariance matrix is

$$\Sigma = \mathbb{E}\big(m_+ m_+'\big) = \mathbb{E}\big((Am + Dv_+)(m'A' + v_+'D')\big) = A\Sigma A' + DD'$$

- here

$$\Sigma = \begin{pmatrix} 25.253 & 3.175 & 19.505 \\ 3.175 & 0.560 & 0.469 \\ 19.505 & 0.469 & 46.550 \end{pmatrix} \varsigma^2.$$

- standard deviation of employment is $\varsigma\sqrt{0.560} = 0.004$
- construct other variables $\tilde{m} = \tilde{A}m$
  - for example, linearize labor wedge:

$$\hat{\tau}(s^t) = 1 - \frac{\hat{\gamma}}{1-\alpha}\left(\frac{\tilde{c}(s^t)}{e^{s_t}\tilde{k}(s^t)^\alpha\big(n(s^t) - \theta(s^t)(1 - n(s^t))\big)^{1-\alpha}}\right) n(s^t)$$

Labor Wedge

- detrended data ($\varepsilon = \infty$)

		τ
	s.d.	0.014
c/y	0.010	-0.131
n	0.010	-0.633

- model

		τ
	s.d.	0.021
c/y	0.018	-0.998
n	0.004	0.962

Growth Rates

- compute comovements of i-period growth rates
- $m_{+i} = A^i m + \sum_{j=0}^{i-1} A^j D v_{+(i-j+1)}$
  - proof by induction
- $\mathbb{E}\big((m_{+i} - m)(m_{+i} - m)'\big) = (A^i - I)\Sigma(A^i - I)' + \sum_{j=0}^{i-1} A^j D D' (A^j)'$
  - proof by induction
- $\tilde{m} = \tilde{A} m \Rightarrow \mathbb{E}\big((\tilde{m}_{+i} - \tilde{m})(\tilde{m}_{+i} - \tilde{m})'\big) = \tilde{A}\mathbb{E}\big((m_{+i} - m)(m_{+i} - m)'\big)\tilde{A}'$

Labor Wedge

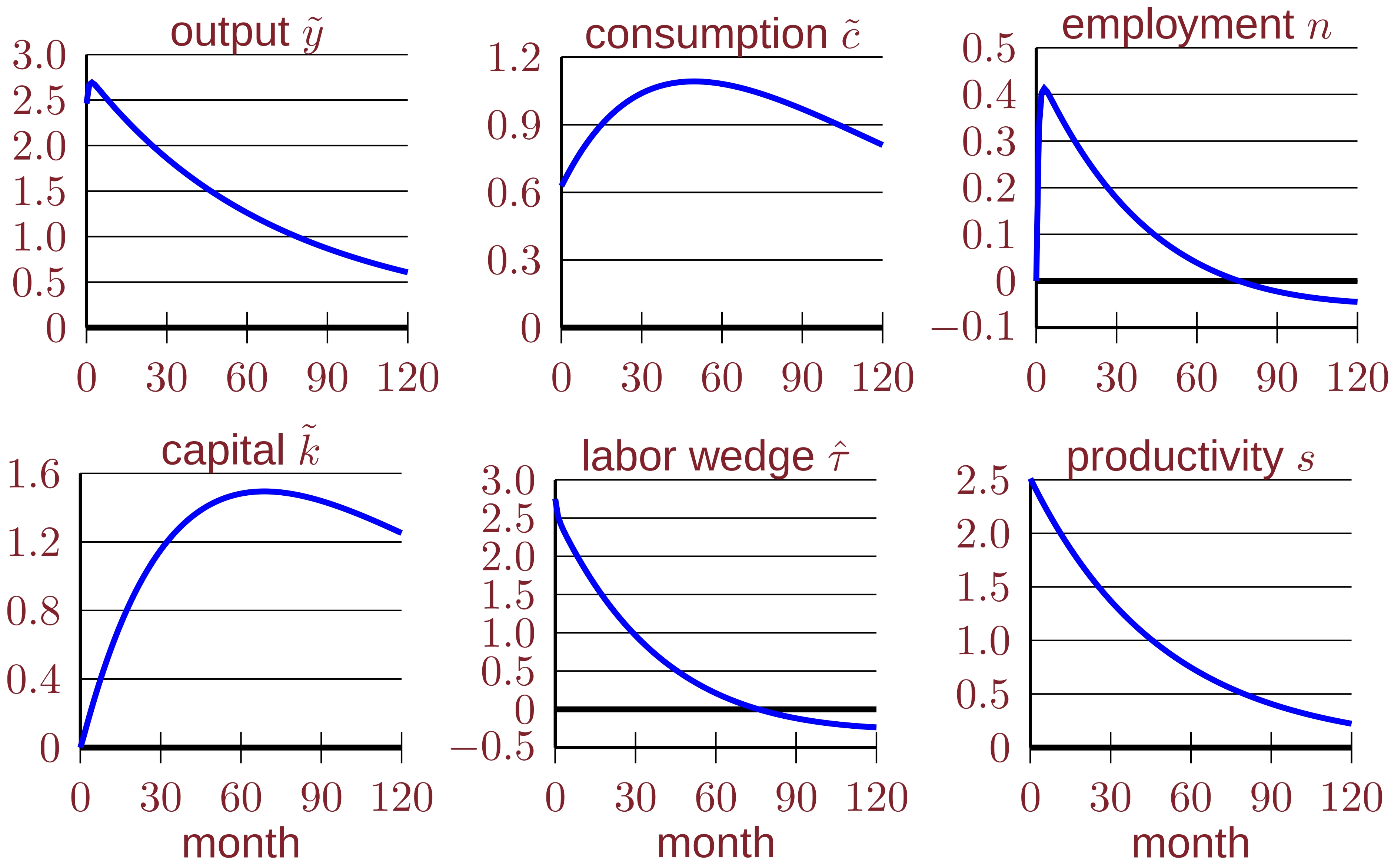
- annual growth rate data ($\varepsilon = \infty$)

		τ
	s.d.	0.023
c/y	0.015	-0.260
n	0.014	-0.597

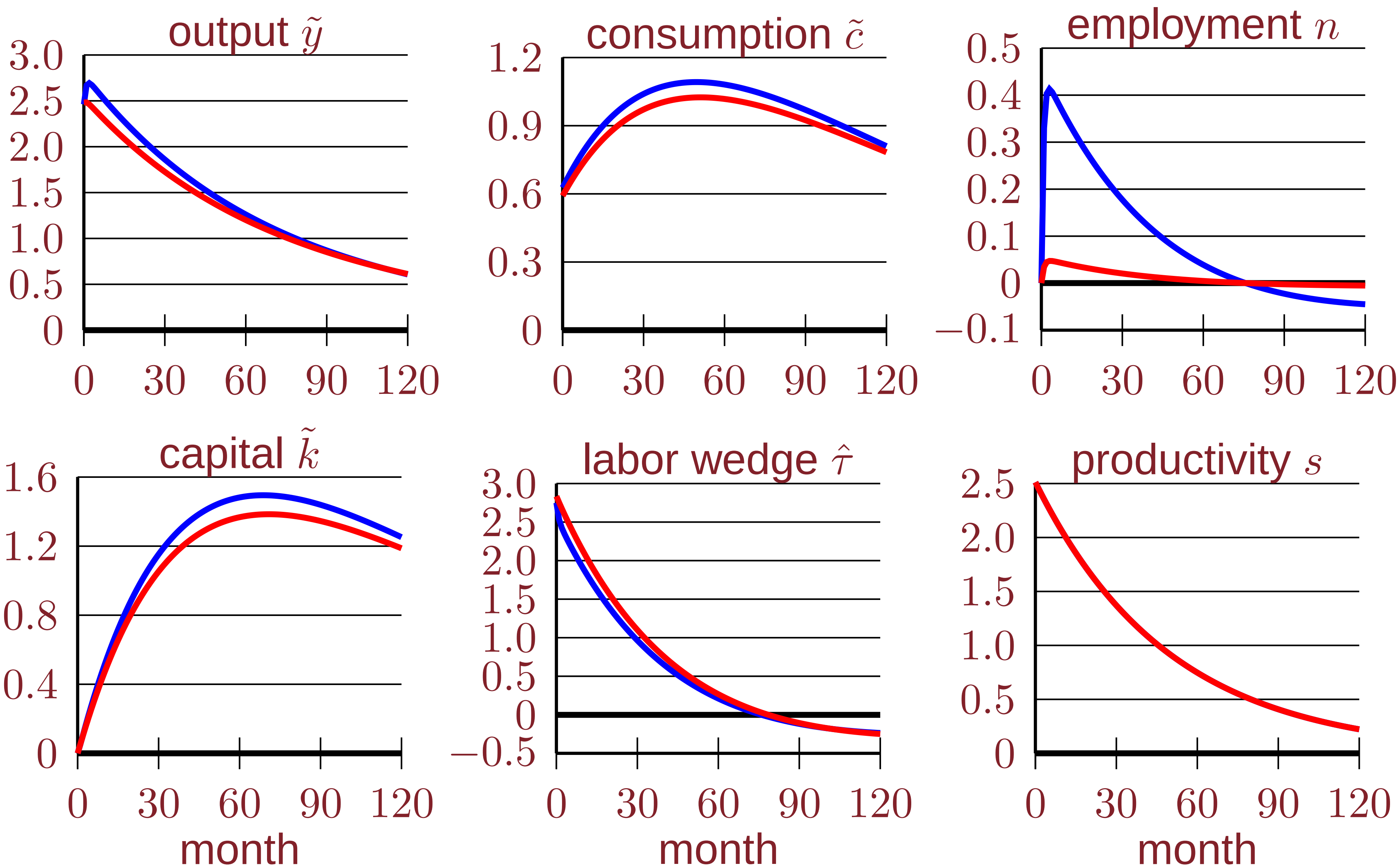
- model

		τ
	s.d.	0.017
c/y	0.014	-0.995
n	0.003	0.876

Impulse Response



Impulse Response



$\bar{\mu} = 2.32$ v.s. $\bar{\mu} = 1$

Stochastic Trend

- suppose $\log z(s^{t+1}) = \log z(s^t) + s_{t+1}$, where s_{t+1} is Markov
- define relative consumption, capital, and wage:
  - $c(s^t) = \tilde{c}(s^t) z(s^t)^{\frac{1}{1-\alpha}}$
  - $k(s^t) = \tilde{k}(s^t) z(s^t)^{\frac{1}{1-\alpha}}$
  - $w(s^t) = \tilde{w}(s^t) z(s^t)^{\frac{1}{1-\alpha}}$

  these three variables are stationary
- so are recruiters/unemployed $\theta(s^t)$ and employment $n(s^t)$

Calibration

- change stochastic process
- $s_{t+1} = 0.0012 + 0.4(s_t - 0.0012) + 0.00325v_{t+1}$
- other calibration targets are unchanged

Log-Linearized System

- policy functions

$$\log(\theta/0.078) = 1.548(s - 0.0012) - 0.480\log(n/0.95) - 2.779\log(\tilde{k}/218.2),$$
$$\log(\tilde{c}/4.696) = 0.381(s - 0.0012) + 0.014\log(n/0.95) + 0.603\log(\tilde{k}/218.2)$$

- state equations

$$\log(n_{+1}/0.95) = 0.026(s - 0.0012) + 0.312\log(n/0.95) - 0.047\log(\tilde{k}/218.2)$$
$$\log(\tilde{k}_{+1}/218.2) = -0.605(s - 0.0012) + 0.019\log(n/0.95) + 0.991\log(\tilde{k}/218.2$$

- eigenvalues 0.99, 0.4, and 0.31
- only response to s changes

Labor Wedge

- detrended data ($\varepsilon = \infty$)

		τ
	s.d.	0.014
c/y	0.010	-0.131
n	0.010	-0.633

- model

		τ
	s.d.	0.009
c/y	0.007	-0.999
n	0.002	0.975

Labor Wedge

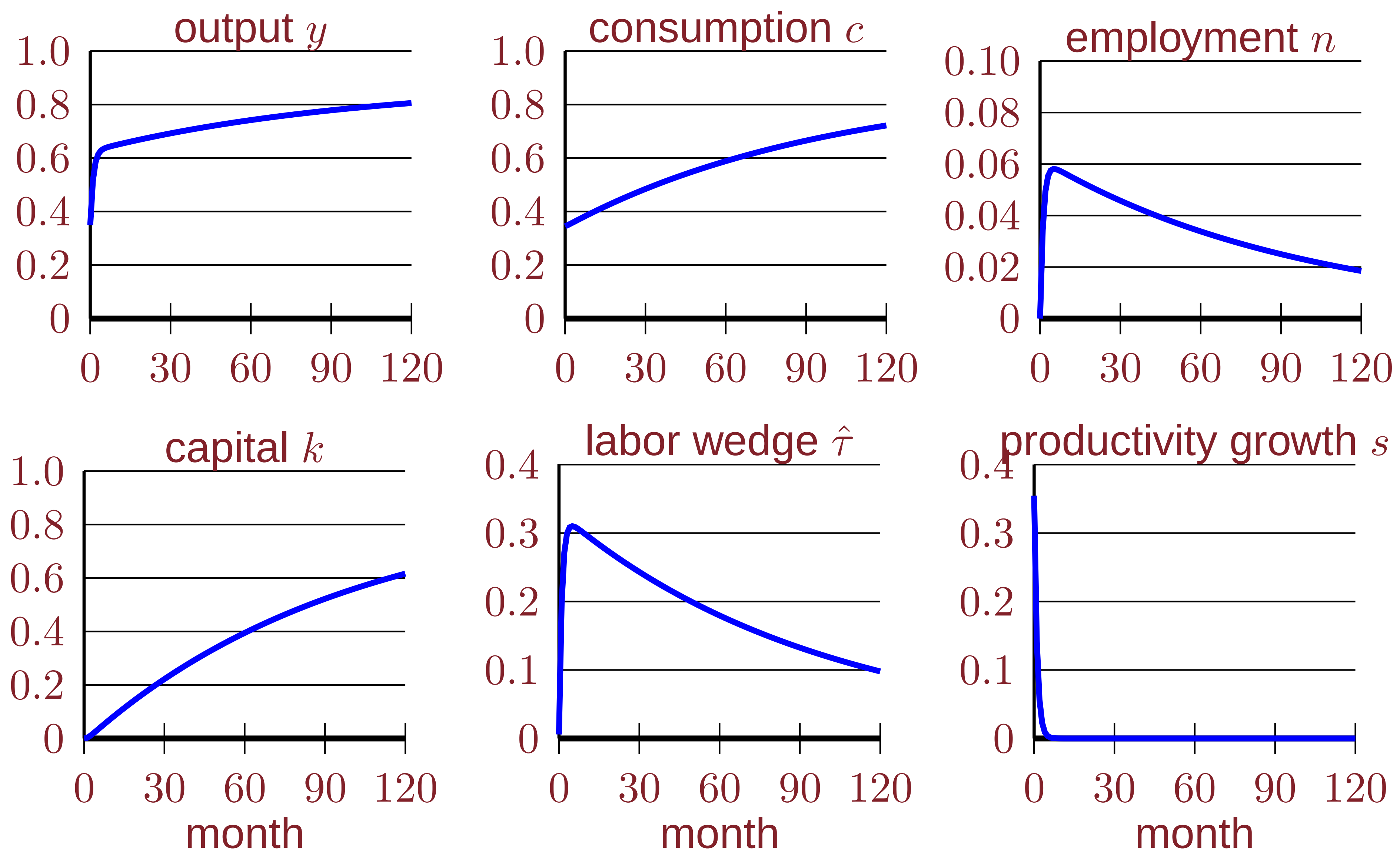
- annual growth rate data ($\varepsilon = \infty$)

		τ
	s.d.	0.023
c/y	0.015	-0.260
n	0.014	-0.597

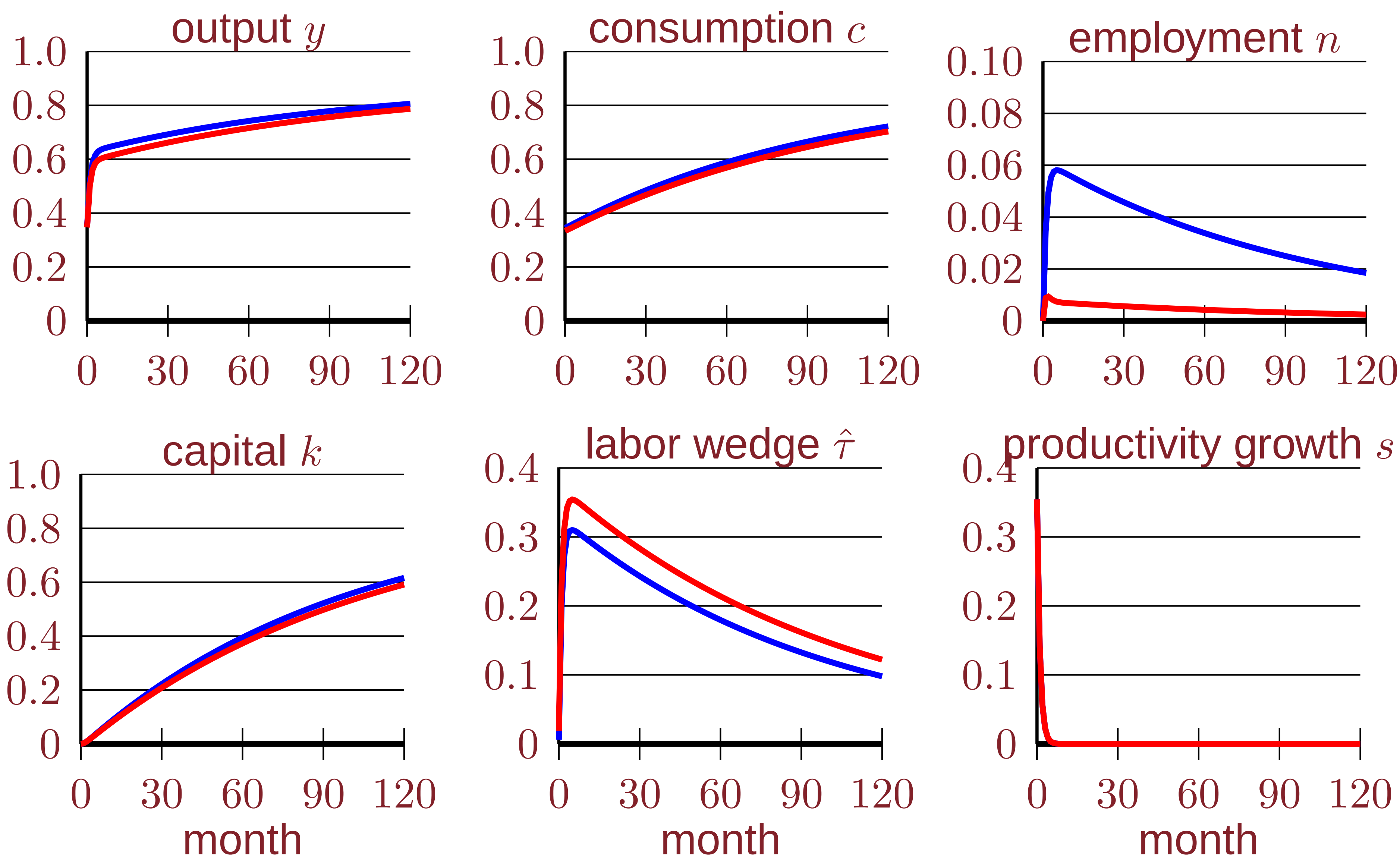
- model

		τ
	s.d.	0.005
c/y	0.004	-0.996
n	0.001	0.851

Impulse Response



Impulse Response



$\bar{\mu} = 2.32$ v.s. $\bar{\mu} = 1$

Other Shocks

- combination of stochastic/deterministic trend
- shocks to the employment exit probability x
- investment-specific technological change
- government spending shocks
- preference shocks and wage markup shocks are off the table

Employment Exit Probability Shocks

- two-shock model, deterministic trend
  - $\log x(s^t) = \log \bar{x} + s_{x,t}$ where $s_{x,t+1} = \rho_x s_{x,t} + \varsigma_x \upsilon_{x,t+1}$
  - $\log z(s^t) = \bar{s}t + s_{z,t}$ where $s_{z,t+1} = \rho_z s_{z,t} + \varsigma_z \upsilon_{z,t+1} - \varsigma_{zx} \upsilon_{x,t+1}$
- calibration:
  - $\rho_x = 0.83$
  - $\varsigma_x = 0.034$
  - $\rho_z = 0.98$
  - $\varsigma_z = 0.0037$
  - $\varsigma_{zx} = 0.0034$: negative correlation between $f(\theta(s^t))$ and $x(s^t)$
- interpret as aggregate and reallocation shock

Labor Wedge

- detrended data ($\varepsilon = \infty$)

		τ
	s.d.	0.014
c/y	0.010	-0.131
n	0.010	-0.633

- model

		τ
	s.d.	0.021
c/y	0.019	-0.988
n	0.006	0.855

Labor Wedge

- annual growth rate data ($\varepsilon = \infty$)

		τ
	s.d.	0.023
c/y	0.015	-0.260
n	0.014	-0.597

- model

		τ
	s.d.	0.016
c/y	0.015	-0.967
n	0.006	0.727

Summary

- with either a deterministic or stochastic trend
  - employment is not very volatile
    - in absolute terms
    - relative to the consumption-output ratio
  - measured labor wedge is positively correlated with employment
- “reallocation shocks” do not change the conclusion

Rigid Wage Model

Wage Indeterminacy

- firm is willing to pay some $w > w(s^t)$, if $\tilde{J}_n(s^t, w) \geq 0$

$$(1-\alpha)z(s^t)\left(\frac{k(s^t)}{n(s^t)-\theta(s^t)(1-n(s^t))}\right)^{\alpha}\left(1+\frac{1-x}{\mu(\theta(s^t))}\right) \geq w.$$

- worker is willing to work at some $w < w(s^t)$, if $\tilde{V}_n(s^t, w) \geq 0$

$$w \geq \frac{\gamma c(s^t)}{1-\tau} - \frac{\phi}{1-\phi}(1-\alpha)z(s^t)\left(\frac{k(s^t)}{n(s^t)-\theta(s^t)(1-n(s^t))}\right)^{\alpha}\left(\frac{1-x-f(\theta(s^t))}{\mu(\theta(s^t))}\right)$$

- this indeterminacy is irrelevant in existing matches
- it is critical for firms’ incentive to recruit
- the wage bands may be quite large
  - balanced growth path: $0.88w(s^t) \leq w \leq 1.12w(s^t)$

Firm Problem

- firm chooses $\{\nu(s^t), n(s^{t+1}), k(s^{t+1})\}$ to maximize

$$\sum_{t=0}^{\infty}\sum_{s^t} q_0(s^t)\Big(z(s^t)k(s^t)^{\alpha}\big(n(s^t)(1-\nu(s^t))\big)^{1-\alpha} + (1-\delta)k(s^t) - k(s^{t+1}) - w(s^t)n(s^t)\Big),$$

where firm growth satisfies

$$n(s^{t+1}) = n(s^t)\big(1 - x + \nu(s^t)\mu(\theta(s^t))\big),$$

taking $k_0 = k(s^0)$, $n_0 = n(s^0)$, and $\{q_0(s^t), w(s^t), \theta(s^t)\}$ as given

Household Problem

- household chooses $\{c(s^t)\}$ to maximize

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\Pi(s^t)\left(\log c(s^t)-\gamma n(s^t)\right)$$

s.t. $a_0=\sum_{t=0}^{\infty}\sum_{s^t}q_0(s^t)\big(c(s^t)-(1-\tau)w(s^t)n(s^t)-T(s^t)\big)$

and $n(s^{t+1})=(1-x)n(s^t)+f(\theta(s^t))(1-n(s^t))$,

taking a_0, $n_0=n(s^0)$, and $\{q_0(s^t),w(s^t),\theta(s^t),\tau,T(s^t)\}$ as given

Government Budget and Market Clearing

- government budget constraint: $T(s^t) = \tau w(s^t) n(s^t)$
- goods market clearing:

$$k(s^{t+1}) = z(s^t)k(s^t)^\alpha \Big(n(s^t)(1-\nu(s^t))\Big)^{1-\alpha} + (1-\delta)k(s^t) - c(s^t)$$

- law of motion for employment:

$$n(s^{t+1}) = (1-x)n(s^t) + f(\theta(s^t))(1-n(s^t))$$

- definition of θ: $\theta(s^t) = \dfrac{\nu(s^t)n(s^t)}{1-n(s^t)}$

Backward-Looking Wages

- target wage:

$$w^*(s^t) = \arg\max_w \tilde{V}_n(s^t, w)^{\phi} \tilde{J}_n(s^t, w)^{1-\phi}$$

  - $\tilde{J}_n(s^t, w)$: value of paying a worker w in s^t, $w(s^{t'})$ thereafter, compared to not employing the worker
  - $\tilde{V}_n(s^t, w)$: value of having a worker paid w in s^t, $w(s^{t'})$ thereafter, compared to having the worker unemployed

Backward-Looking Wages

- target wage:

$$w^*(s^t) = \arg\max_w \tilde{V}_n(s^t, w)^{\phi} \tilde{J}_n(s^t, w)^{1-\phi}$$

  - $\tilde{J}_n(s^t, w)$: value of paying a worker w in s^t, $w(s^{t'})$ thereafter, compared to not employing the worker
  - $\tilde{V}_n(s^t, w)$: value of having a worker paid w in s^t, $w(s^{t'})$ thereafter, compared to having the worker unemployed

- actual wage: $w(s^t) = rw(s^{t-1})e^{\frac{\bar{s}}{1-\alpha}} + (1-r)w^*(s^t)$
  - $r \in [0, 1]$ indicates extent of wage rigidity
  - $\bar{s}$ is average productivity growth

Wage Behavior

- forward-looking equation for the target wage:

$$w^*(s^t) = \phi(1-\alpha)z(s^t)\left(\frac{k(s^t)}{n(s^t)(1-\nu(s^t))}\right)^{\alpha}(1+\theta(s^t)) + (1-\phi)\frac{\gamma c(s^t)}{1-\tau}$$
$$+ (1 - x - f(\theta(s^t))) \sum_{s^{t+1}|s^t} q_t(s^{t+1})\left(w^*(s^{t+1}) - w(s^{t+1})\right)$$

- backward-looking equation for the actual wage:

$$w(s^t) = rw(s^{t-1})e^{\frac{\bar{s}}{1-\alpha}} + (1-r)w^*(s^t)$$

- saddle-path dynamics, downward-sloping saddle path

Balanced Growth

- suppose $\log z(s^{t+1}) = \log z(s^t) + \bar{s}$
- consumption, capital, and actual and target wages grow at rate $\dfrac{\bar{s}}{1-\alpha}$
  - $c(s^t) = \bar{c} z(s^t)^{\frac{1}{1-\alpha}}$
  - $k(s^t) = \bar{k} z(s^t)^{\frac{1}{1-\alpha}}$
  - $w(s^t) = \bar{w} z(s^t)^{\frac{1}{1-\alpha}}$
  - $w^*(s^t) = \bar{w}^* z(s^t)^{\frac{1}{1-\alpha}}$
- recruiters/unemployed and employment are constant:
  - $\theta(s^t) = \bar{\theta}$
  - $n(s^t) = \bar{n}$
- this implies $w(s^t) = w^*(s^t)$, so no distortions from rigidity

Deterministic Trend

Calibration

- new parameter is wage rigidity $r = 0.95$
- other parameters are unchanged

Policy Functions

- linearize system around steady state

$$\begin{aligned}
\log\theta &= \log 0.078 + 40.825s - 0.630\log(n/0.95) \\
&\qquad + 10.441\log(\tilde{k}/218.2) - 38.184\log(\tilde{w}_{-1}/4.017), \\
\log\tilde{c} &= \log 4.696 + 0.259s + 0.014\log(n/0.95) \\
&\qquad + 0.607\log(\tilde{k}/218.2) - 0.023\log(\tilde{w}_{-1}/4.017), \\
\log\tilde{w}^* &= \log 4.017 + 2.974s - 0.215\log(n/0.95) \\
&\qquad + 1.146\log(\tilde{k}/218.2) - 2.321\log(\tilde{w}_{-1}/4.017).
\end{aligned}$$

- compare to flexible wage model ($r = 0$)

$$\begin{aligned}
\log\theta &= \log 0.078 + 7.387s - 0.480\log(n/0.95) - 2.779\log(\tilde{k}/218.2), \\
\log\tilde{c} &= \log 4.696 + 0.250s + 0.014\log(n/0.95) + 0.603\log(\tilde{k}/218.2)
\end{aligned}$$

State Equations

- rigid wages ($r = 0.95$)

$$\begin{aligned}
\log n_{+1} &= \log 0.95 + 0.694s + 0.309\log(n/0.95) \\
&\qquad + 0.178\log(\tilde{k}/218.2) - 0.649\log(\tilde{w}_{-1}/4.017), \\
\log \tilde{k}_{+1} &= \log 218.2 + 0.018s + 0.019\log(n/0.95) \\
&\qquad + 0.990\log(\tilde{k}/218.2) + 0.003\log(\tilde{w}_{-1}/4.017), \\
\log \tilde{w} &= \log 4.107 + 0.149s - 0.011\log(n/0.95) \\
&\qquad + 0.057\log(\tilde{k}/218.2) + 0.834\log(\tilde{w}_{-1}/4.017).
\end{aligned}$$

- compare to flexible wage model ($r = 0$)

$$\begin{aligned}
\log n_{+1} &= \log 0.95 + 0.126s + 0.312\log(n/0.95) - 0.047\log(\tilde{k}/218.2), \\
\log \tilde{k}_{+1} &= \log 218.2 + 0.020s + 0.019\log(n/0.95) + 0.991\log(\tilde{k}/218.2)
\end{aligned}$$

State Equations

- rigid wages ($r = 0.95$)

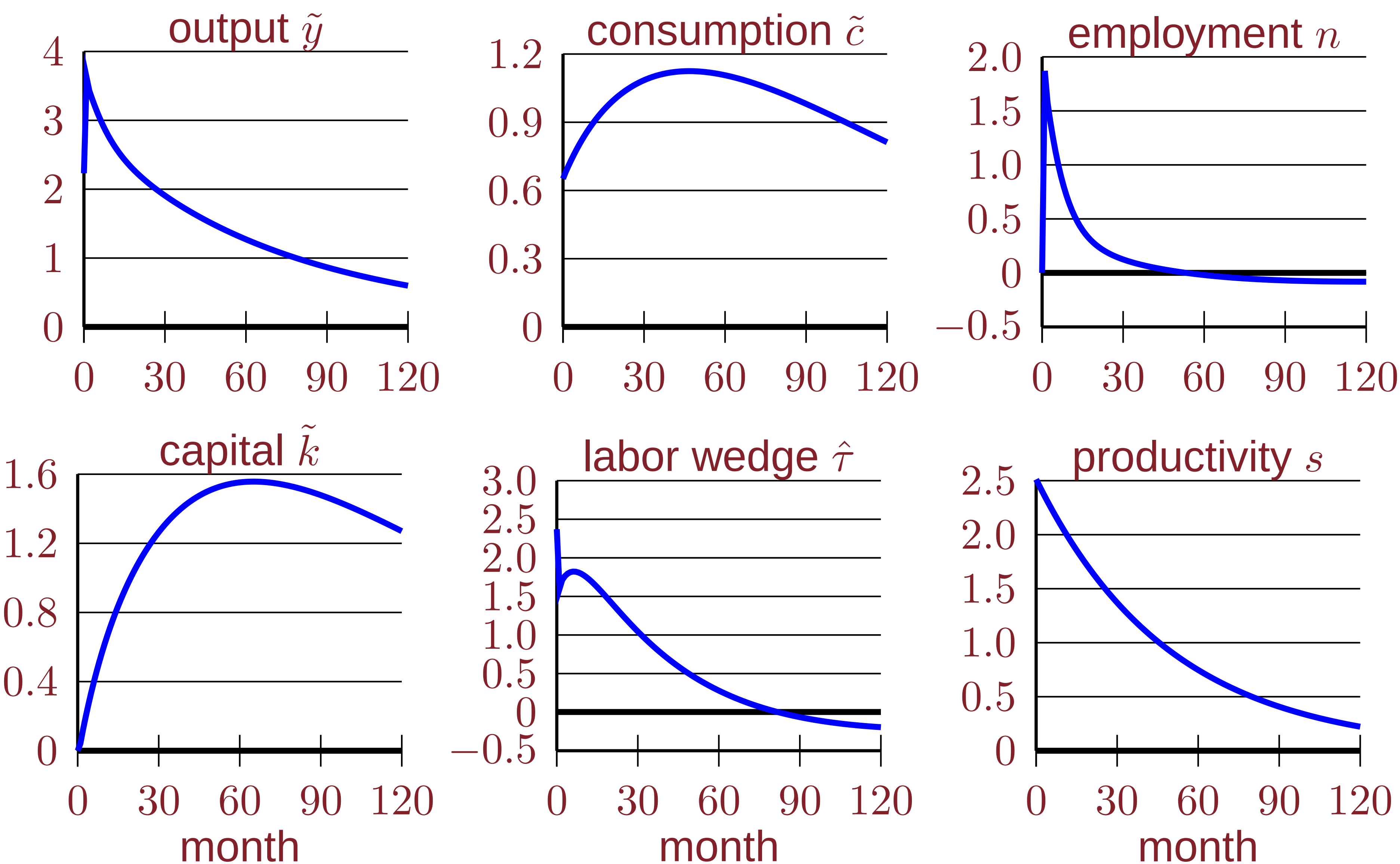
$$\begin{aligned}
\log n_{+1} &= \log 0.95 + 0.694 s + 0.309 \log(n/0.95) \\
&\qquad + 0.178 \log(\tilde{k}/218.2) - 0.649 \log(\tilde{w}_{-1}/4.017), \\
\log \tilde{k}_{+1} &= \log 218.2 + 0.018 s + 0.019 \log(n/0.95) \\
&\qquad + 0.990 \log(\tilde{k}/218.2) + 0.003 \log(\tilde{w}_{-1}/4.017), \\
\log \tilde{w} &= \log 4.107 + 0.149 s - 0.011 \log(n/0.95) \\
&\qquad + 0.057 \log(\tilde{k}/218.2) + 0.834 \log(\tilde{w}_{-1}/4.017).
\end{aligned}$$

- compare to flexible wage model ($r = 0$)

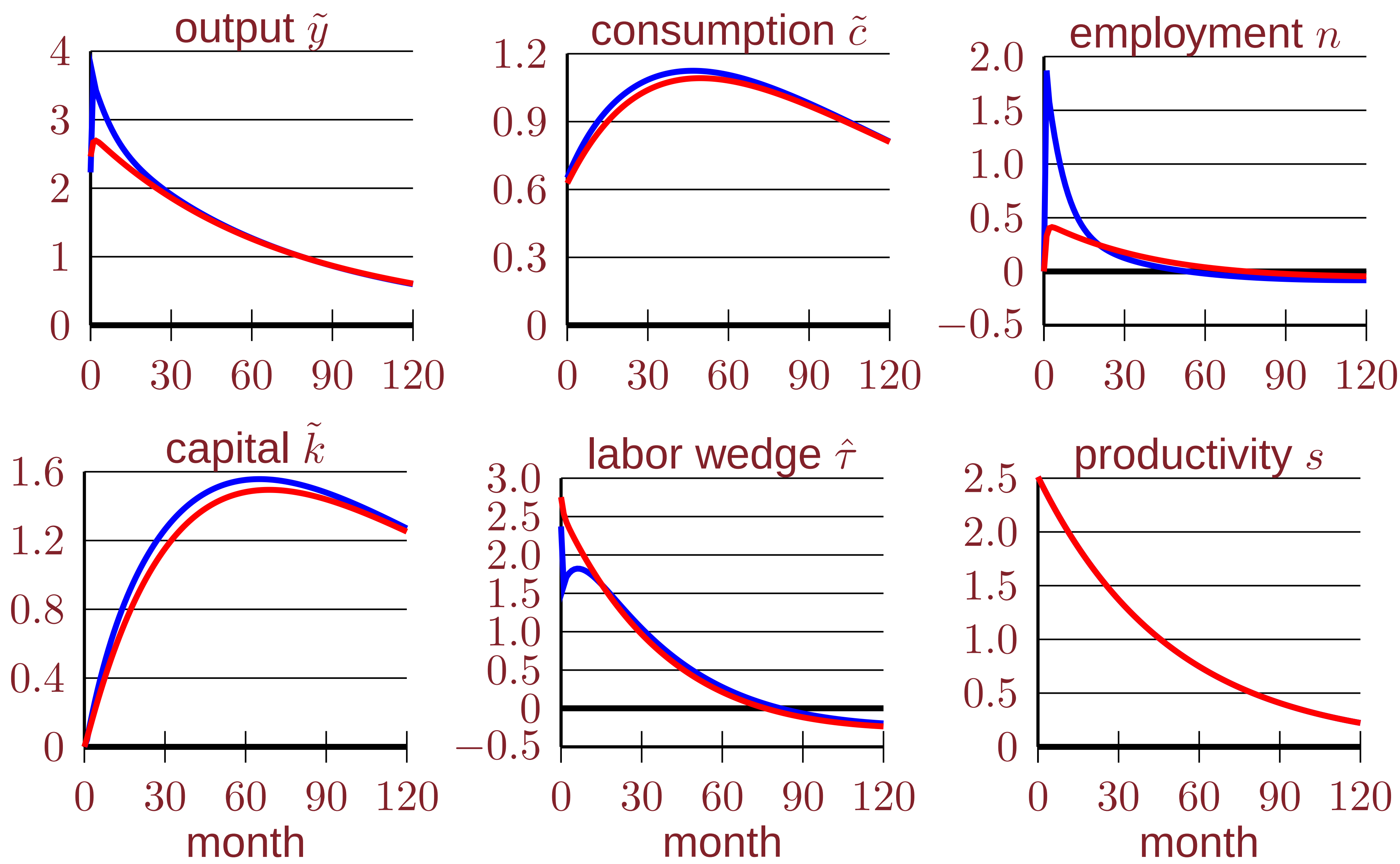
$$\begin{aligned}
\log n_{+1} &= \log 0.95 + 0.126 s + 0.312 \log(n/0.95) - 0.047 \log(\tilde{k}/218.2), \\
\log \tilde{k}_{+1} &= \log 218.2 + 0.020 s + 0.019 \log(n/0.95) + 0.991 \log(\tilde{k}/218.2)
\end{aligned}$$

- eigenvalues $k : 0.99$, $s : 0.98$, $\tilde{w} : 0.85$, and $n : 0.29$.

Impulse Response



Impulse Response



$r = 0.95$ v.s. $r = 0$

Labor Wedge

- detrended data ($\varepsilon = \infty$)

		τ
	s.d.	0.014
c/y	0.010	-0.131
n	0.010	-0.633

- model

		τ
	s.d.	0.019
c/y	0.020	-0.938
n	0.010	0.690

Labor Wedge

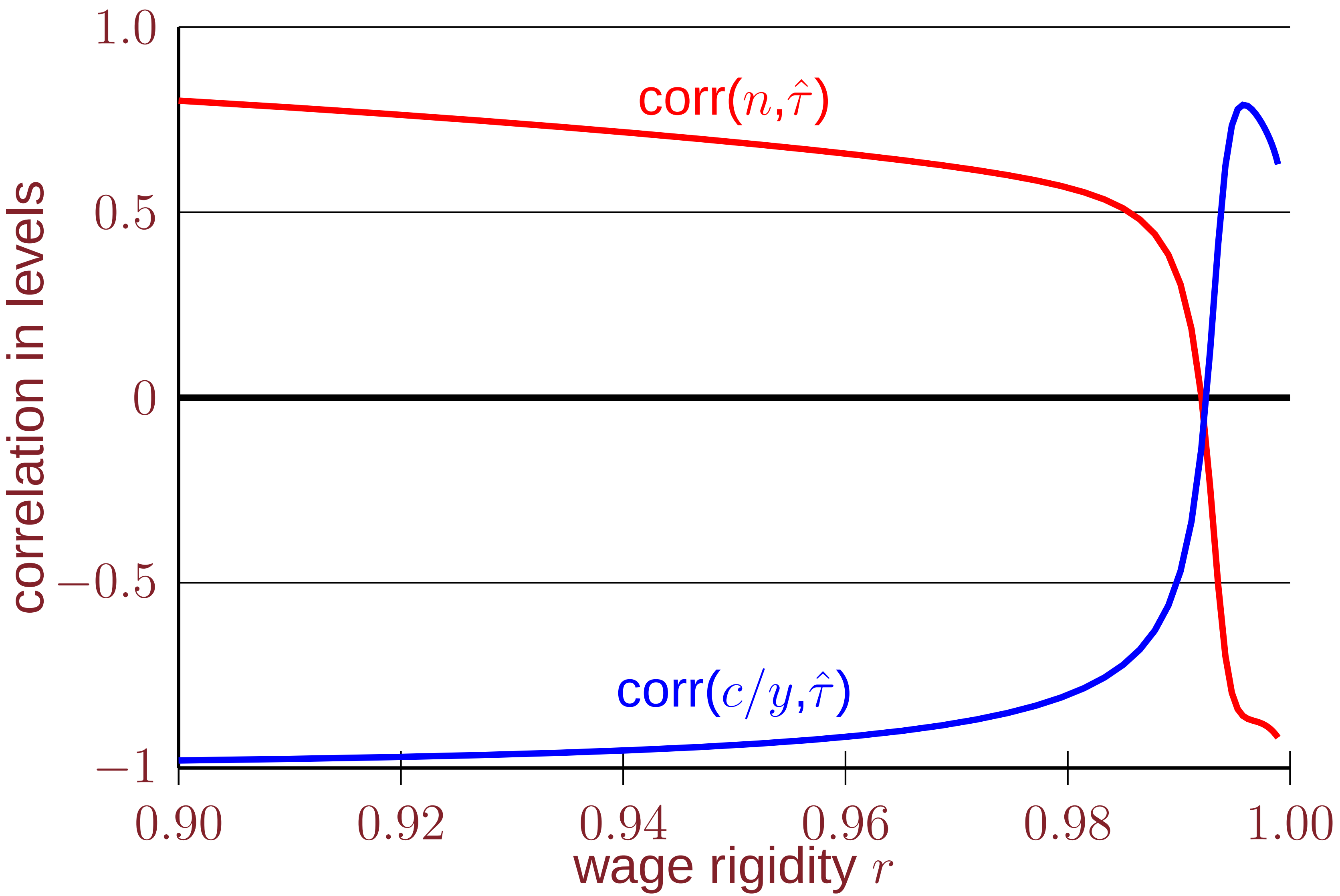
- annual growth rate data ($\varepsilon = \infty$)

		τ
	s.d.	0.023
c/y	0.015	-0.260
n	0.014	-0.597

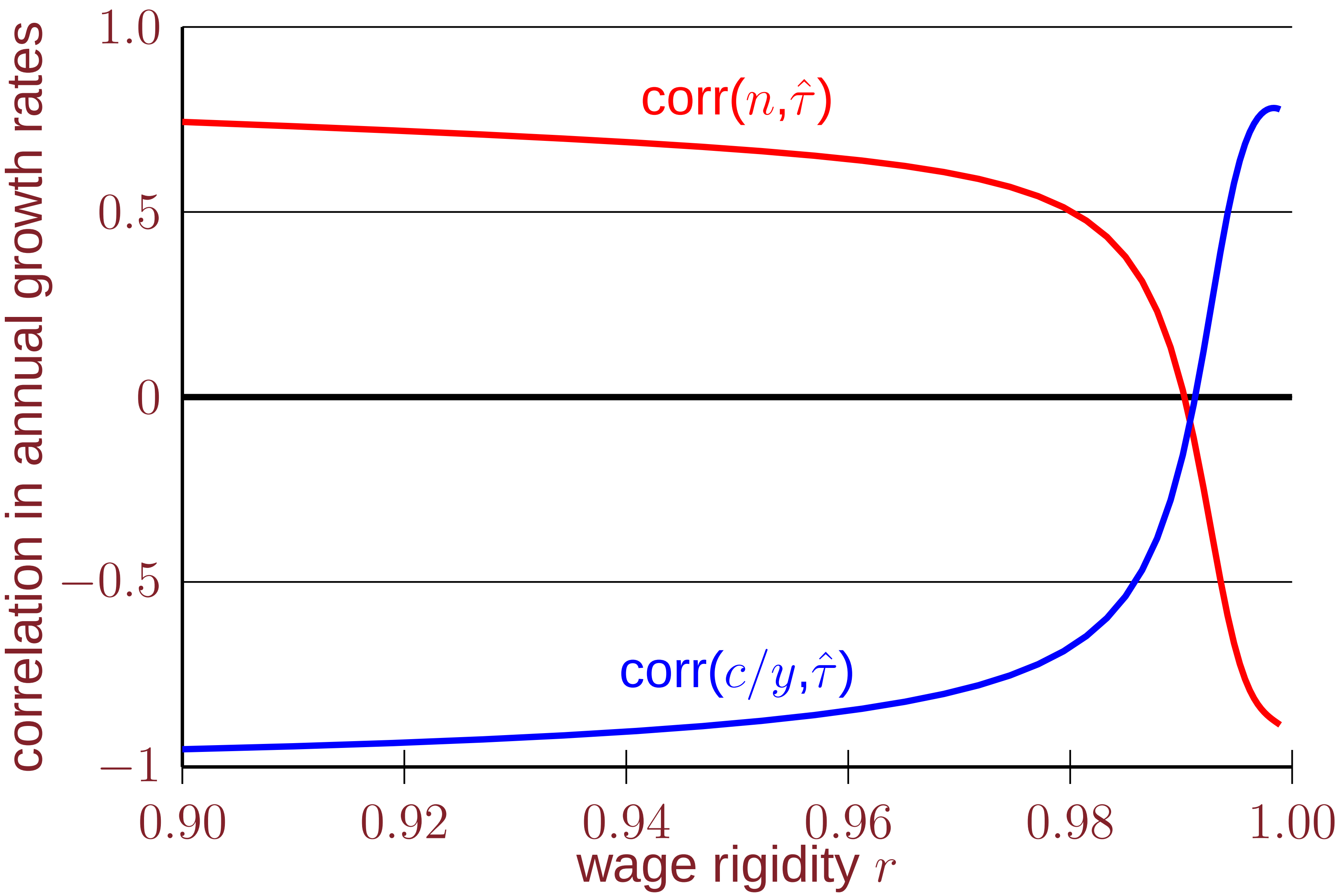
- model

		τ
	s.d.	0.013
c/y	0.019	-0.881
n	0.012	0.670

Sensitivity to Rigidity r



Sensitivity to Rigidity r



Stochastic Trend

Calibration

- new parameter is wage rigidity $r = 0.95$
- other parameters are unchanged

Policy Functions

- linearize system around steady state

$$\log(\theta/0.078) = 81.417(s - 0.0012) - 0.630\log(n/0.95) + 10.441\log(\tilde{k}/218.2) - 38.184\log(\tilde{w}_{-1}/4.017),$$

$$\log(\tilde{c}/4.696) = 0.436(s - 0.0012) + 0.014\log(n/0.95) + 0.607\log(\tilde{k}/218.2) - 0.023\log(\tilde{w}_{-1}/4.017),$$

$$\log(\tilde{w}^*/4.017) = 5.096(s - 0.0012) - 0.215\log(n/0.95) + 1.146\log(\tilde{k}/218.2) - 2.321\log(\tilde{w}_{-1}/4.017).$$

- compare to flexible wage model ($r = 0$)

$$\log(\theta/0.078) = 1.548(s - 0.0012) - 0.480\log(n/0.95) - 2.779\log(\tilde{k}/218.2),$$

$$\log(\tilde{c}/4.696) = 0.381(s - 0.0012) + 0.014\log(n/0.95) + 0.603\log(\tilde{k}/218.2)$$

State Equations

- rigid wages ($r = 0.95$)

$$
\begin{aligned}
\log(n_{+1}/0.95) &= 1.384(s - 0.0012) + 0.309\log(n/0.95) \\
&\qquad + 0.178\log(\tilde{k}/218.2) - 0.649\log(\tilde{w}_{-1}/4.017), \\
\log(\tilde{k}_{+1}/218.2) &= -0.612(s - 0.0012) + 0.019\log(n/0.95) \\
&\qquad + 0.990\log(\tilde{k}/218.2) + 0.003\log(\tilde{w}_{-1}/4.017), \\
\log(\tilde{w}/4.017) &= -1.163(s - 0.0012) - 0.011\log(n/0.95) \\
&\qquad + 0.057\log(\tilde{k}/218.2) + 0.834\log(\tilde{w}_{-1}/4.017).
\end{aligned}
$$

- compare to flexible wage model ($r = 0$)

$$
\begin{aligned}
\log(n_{+1}/0.95) &= 0.026(s - 0.0012) + 0.312\log(n/0.95) - 0.047\log(\tilde{k}/218.2) \\
\log(\tilde{k}_{+1}/218.2) &= -0.605(s - 0.0012) + 0.019\log(n/0.95) + 0.991\log(\tilde{k}/218.2
\end{aligned}
$$

State Equations

- rigid wages ($r = 0.95$)

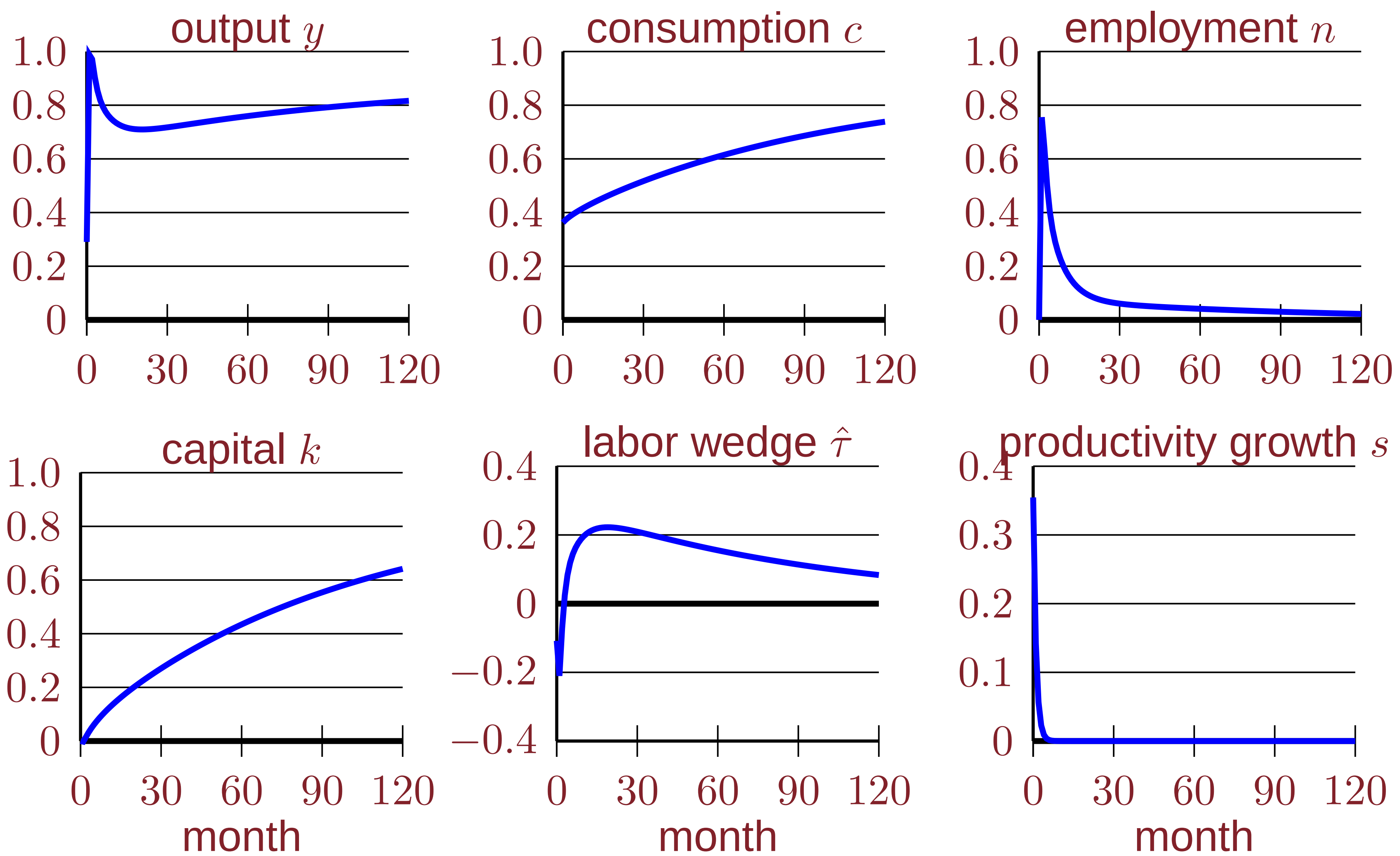
$$
\begin{aligned}
\log(n_{+1}/0.95) &= 1.384(s - 0.0012) + 0.309 \log(n/0.95) \\
&\quad + 0.178 \log(\tilde{k}/218.2) - 0.649 \log(\tilde{w}_{-1}/4.017), \\
\log(\tilde{k}_{+1}/218.2) &= -0.612(s - 0.0012) + 0.019 \log(n/0.95) \\
&\quad + 0.990 \log(\tilde{k}/218.2) + 0.003 \log(\tilde{w}_{-1}/4.017), \\
\log(\tilde{w}/4.017) &= -1.163(s - 0.0012) - 0.011 \log(n/0.95) \\
&\quad + 0.057 \log(\tilde{k}/218.2) + 0.834 \log(\tilde{w}_{-1}/4.017).
\end{aligned}
$$

- compare to flexible wage model ($r = 0$)

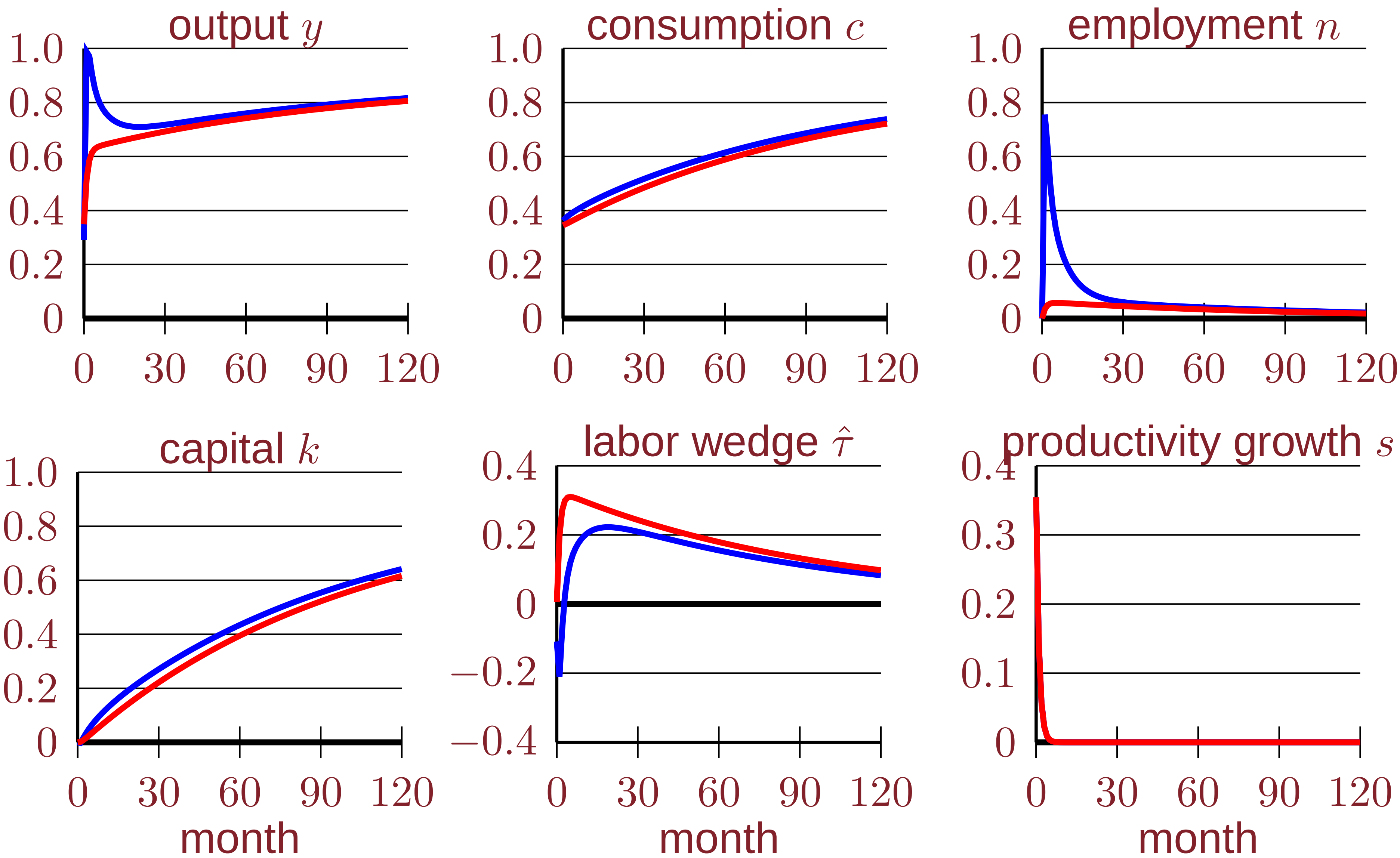
$$
\begin{aligned}
\log(n_{+1}/0.95) &= 0.026(s - 0.0012) + 0.312 \log(n/0.95) - 0.047 \log(\tilde{k}/218.2) \\
\log(\tilde{k}_{+1}/218.2) &= -0.605(s - 0.0012) + 0.019 \log(n/0.95) + 0.991 \log(\tilde{k}/218.2
\end{aligned}
$$

- eigenvalues $k : 0.99$, $s : 0.98$, $\tilde{w} : 0.85$, and $n : 0.29$.

Impulse Response



Impulse Response



$r = 0.95$ v.s. $r = 0$

Labor Wedge

- detrended data ($\varepsilon = \infty$)

		τ
	s.d.	0.014
c/y	0.010	-0.131
n	0.010	-0.633

- model

		τ
	s.d.	0.008
c/y	0.011	0.040
n	0.013	-0.449

Labor Wedge

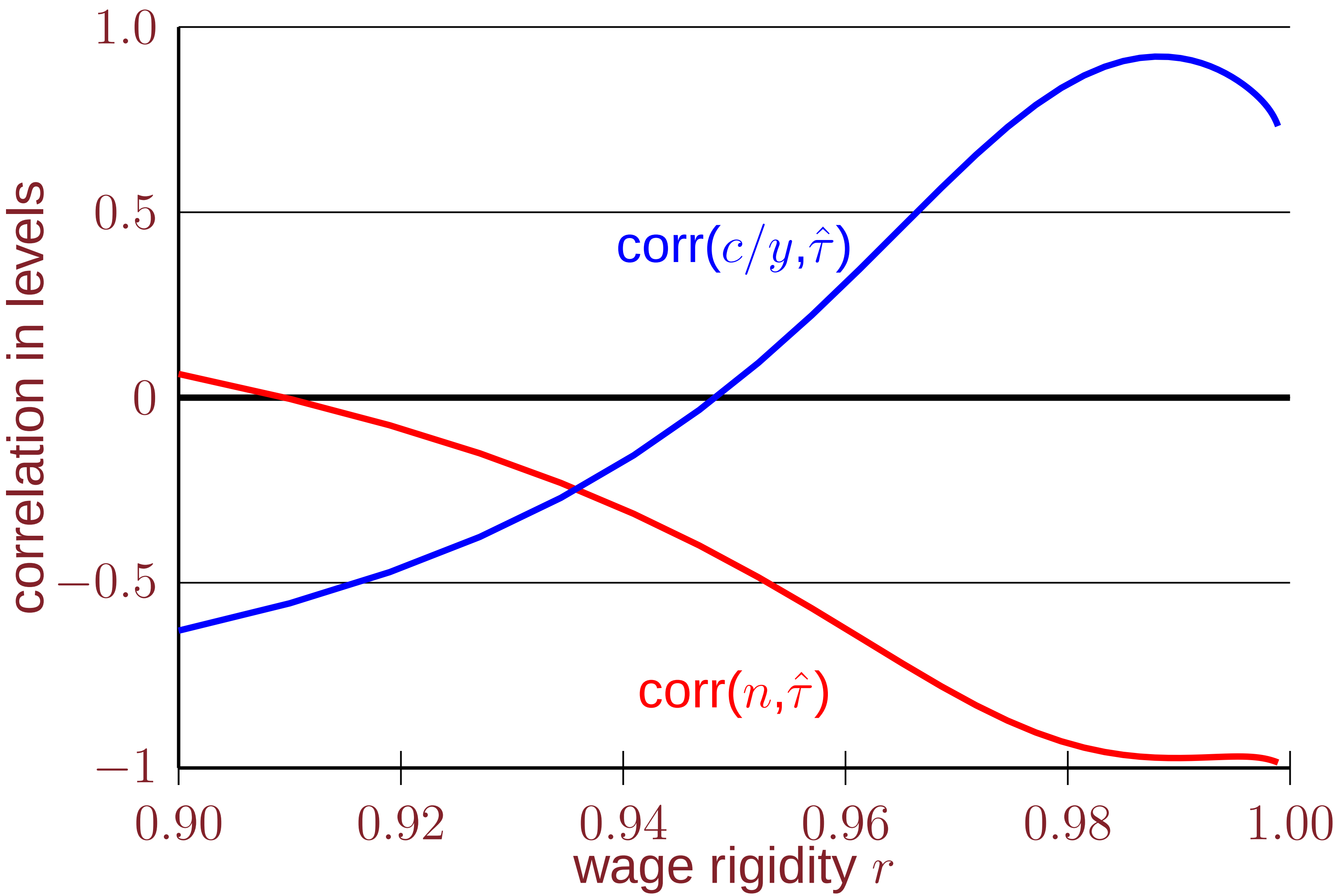
- annual growth rate data ($\varepsilon = \infty$)

		τ
	s.d.	0.023
c/y	0.015	-0.260
n	0.014	-0.597

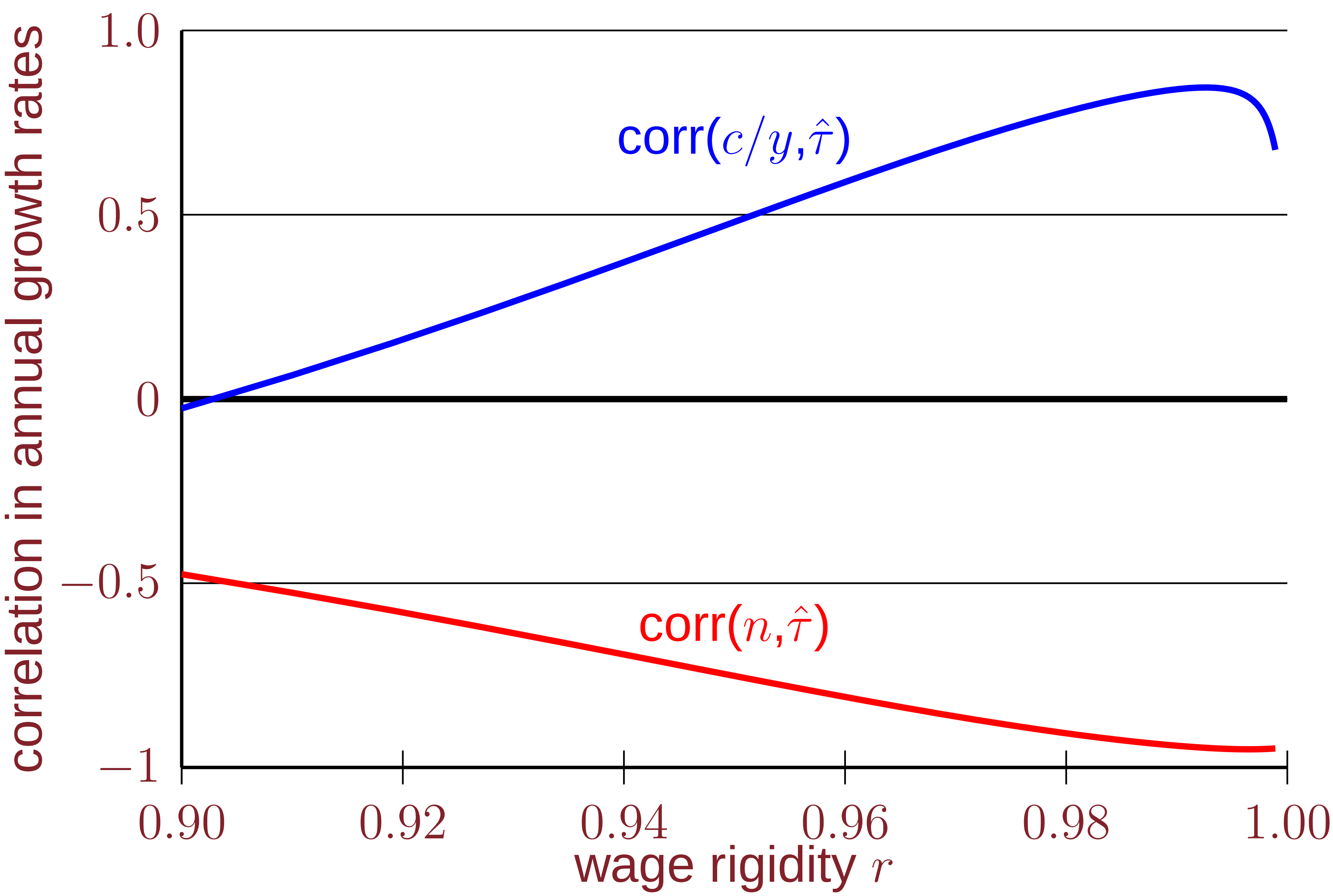
- model

		τ
	s.d.	0.009
c/y	0.012	0.481
n	0.016	-0.751

Sensitivity to Rigidity r



Sensitivity to Rigidity r



Looking Ahead

Speculation

- testable predictions (hours margin)
- finite sample properties of the model
- policy analysis and exploration of other shocks (gov't spending)
- microfoundations of bargaining (Hall-Milgrom, Gertler-Trigari)
- micro-measurement of wage rigidity (Pissarides, Haefke et al)
- alternatives to search frictions (mismatch)
- other markets (housing, financial markets)