Age Effects and the Pre-Sample Evolution of Income and Consumption Inequality

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1 Introduction

In an earlier version of our paper "Heterogeneous Life-Cycle Profiles, Income Risk and Consumption Inequality",[1] we used the age effects in income and consumption inequality to estimate the pre-sample sources of inequality. Those estimates were based on incorrectly derived moment conditions. The correct moment conditions do not separately identify the different sources of pre-sample inequality. Therefore, the latest version of the paper no longer includes these estimates.

In this note we (i) present the correct derivation of the moment conditions for differences in inequality across age, and (ii) explain the identification problem. Before reading this note, it is necessary to read the paper up to section 4.1 (Moment conditions).

2 Age effects in inequality

Because our dataset contains 5 cohorts (at least 3 in every year), we can take first differences across cohorts as well as over time. Under the model, the age effects in inequality are informative about the sources of inequality in the pre-sample period. To see this, consider the evolution of consumption inequality, explicitly taking into account that each year the cohort grows a year older as well.

$$var_{at}\left(c\right) = var_{a-1,t-1}\left(c\right) + var_{t}\left(v\right) \tag{1}$$

where a is age. Iterating back this expression to the 'birth year' of the cohort (the year in which the cohort entered the labor force), we get

$$var_{at}\left(c\right) = var_{0,t-a}\left(c\right) + \sum_{s=0}^{a-1} var_{t-s}\left(v\right) \tag{2}$$

[1] Versions from August 2007 and before. These earlier versions are still available as CEPR discussion paper 5881 or IZA discussion paper 3239.

Now consider two cohorts in year t, one with age a and another one with age $a-10$. Then, taking a first difference across cohorts, we get the following moment condition.

$$\Delta_a var_{at}(c) = var_{at}(c) - var_{a-10,t}(c) \tag{3}$$

$$= \Delta_a var_{0,t-a}(c) + \sum_{s=a-10}^{a-1} var_{t-s}(v) \tag{4}$$

Inequality between the two cohorts differs, because of cohort effects (the difference between the cohorts' initial inequality when they enter the labor market) and because of shocks that were realized between $t-a+10$ and $t-a$, when the older cohort was already alive but the younger one was not yet.

3 Cohort effects in consumption inequality

In the earlier versions of our paper, we wrote that equation (4) and similar moment conditions for $\Delta_a var_{at}(y)$ and $\Delta_a cov_{at}(y,c)$ could be used to identify $var_t(v)$ and $var_t(\alpha)$ prior to our sample period. To do this, we assumed that there are no cohort effects in income and consumption inequality: all cohorts start with the same inequality at birth so that $\Delta_a var_{0,t-a}(c) = var_{0,t-a}(c) - var_{0,t-a+10}(c) = 0$.[2] Whereas this assumption is reasonable (although untestable) for income inequality, for consumption inequality it is inconstent with the model.[3]

To derive the cohort effects in consumption inequality, recall the model (abstracting from transitory shocks for simplicity),

$$y_{i,a,t} = y_{i,a-1,t-1} + \alpha_{i,t} + v_{i,t} \tag{5}$$

$$c_{i,a,t} = c_{i,a-1,t-1} + v_{i,t} \tag{6}$$

which implies that

$$y_{i,a,t} = y_{i,0,t-a} + \sum_{s=0}^{a-1} (\alpha_{i,t-s} + v_{i,t-s}) \tag{7}$$

$$c_{i,a,t} = c_{i,0,t-a} + \sum_{s=0}^{a-1} v_{i,t-s} \tag{8}$$

Here, $y_{i,0,t-a}$ and $c_{i,0,t-a}$ are income and consumption of individual i at birth in time period $t-a$. The crucial thing is that while $y_{i,0,t-a}$ may be the same

[2] In addition, we needed the assumption that the contribution to inequality due to transitory shocks in the pre-sample period averages out so that we could set it to zero. This assumption reflects the fact that, for large age differences, differences in inequality across cohorts are caused only by permanent shocks because the contribution of permanent shocks cumulates over time whereas the contribution of transitory shocks does not.

[3] We are grateful to Claudio Michelacci for drawing our attention to this mistake.

for all individuals i or may even be equal to zero, $c_{i,0,t-a}$ cannot be the same for everyone.

The value of $c_{i,0,t-a}$ is determined by the transversality condition, or the fact that the IBC holds with equality. With a non-stochastic, constant interest rate $R = 1+r$, the IBC for a consumer born at time t can be written as follows.

$$\sum_{a=0}^{\infty} R^{-a} c_{i,a,t+a} = \sum_{a=0}^{\infty} R^{-a} y_{i,a,t+a} \tag{9}$$

Using the expressions above this implies an expression for consumption at birth.

$$c_{i,0,t} = y_{i,0,t} + \frac{R-1}{R} \sum_{a=0}^{\infty} R^{-a} \left(\sum_{s=0}^{a-1} \alpha_{i,t+a-s} \right). \tag{10}$$

Working out the double sum, this equation simplifies to the following.

$$c_{i,0,t} = y_{i,0,t} + \frac{R-1}{R} \sum_{s=1}^{\infty} \left(\sum_{j=s}^{\infty} \frac{1}{R^j} \right) \alpha_{i,t+s} = y_{i,0,t} + \sum_{s=1}^{\infty} R^{-s} \alpha_{i,t+s} \tag{11}$$

Since each individual will receive different predictable shocks, consumption smoothing implies that each individual starts off at a different level of consumption.

Calculating a cross-sectional variance from this expression, setting the interest rate to zero, $R = 1$, like we do in the paper and assuming no cohort effects in income, i.e. $var_{0,t-a}(y) = var_{0,t-a+10}(y)$, we obtain an expression for the differences in consumption inequality at the time when a cohort enters the labor market.

$$\Delta_a var_{0,t-a}(c) = \sum_{s=1}^{10} var_{t-a+s}(\alpha) = \sum_{s=a-10}^{a-1} var_{t-s}(\alpha) \tag{12}$$

The initial difference in the dispersion of consumption between two cohorts equals the sum of all the trend shocks that one cohort gets and the other does not. These shocks are just the shocks when the older cohort is already born and the younger one is not born yet. There are also some shocks when the older cohort is already dead while the younger cohort is still alive, but because our cohorts are infinitely lived, these shocks do not matter.

4 Identifying pre-sample sources of inequality

Having derived a model-consistent expression for the cohort effect in consumption inequality, we can use (4) to derive expressions for the differences in consumption inequality across cohorts of different ages in the same year,

$$\Delta_a var_{at}(c) = \sum_{s=a-10}^{a-1} \left[var_{t-s}(\alpha) + var_{t-s}(v) \right] = 10\,(V + A) \tag{13}$$

where the second equality follows from assuming that the variances of α_i and v_i are constant in the pre-sample period, $var_t(\alpha) = A$ and $var_t(v) = V$ for all $t < t_0$, which is purely for simplicity.

From (4), it is immediate that A and V are not separately identified from eachother, becaus $\Delta_a var_{at}(c) = \Delta_a var_{at}(y)$. As usual, the covariance between income and consumption does not contain independent information either.

$$\Delta_a cov_{at}(y, c) = \Delta_a cov(y_{i,a,t}, c_{i,a,t}) = 10(V + A) \tag{14}$$

Thus, for the sample period, we can separately identify $var_t(\alpha)$ and $var_t(v)$ using the comovement of consumption and income, but for the pre-sample period we cannot.